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The open superstring 5-point amplitude revisited

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Abstract: We derive the complete five-gluon scattering amplitude at tree level, within the context of Open Superstring theory. We find the general expression in terms of kinematic factors, and also find its complete expansion up to $\mathcal{O}(\alpha'^3)$ terms. We use our scattering amplitude to test three non-equivalent $\mathcal{O}(\alpha'^3)$ effective lagrangians that have recently been matter of some controversy.

Keywords: Superstrings and Heterotic Strings, D-branes.

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Contents

1. Introduction

A nice feature of String Theory, among many things, is the fact that it reproduces known non-massive field theories in its low energy limit ($\alpha' \to 0$). In fact, the String Theory corrections to these field theories may be found perturbatively in α' by means of scattering amplitude arguments, as was done in [1], where the leading corrections to the Yang-Mills

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and the Einstein-Hilbert lagrangians were found. Moreover, in [2, 3], the infinite α' series describing the interaction between open massless strings (corresponding to photons) was found to be the Born-Infeld theory [4], as long as the field strength $F_{\mu\nu}$ is kept constant.

Up to now, there hasn't been found a non-abelian generalization of this last result and the only achievements that have been done so far are strictly perturbative in α'. Indeed, besides the leading Yang-Mills term, the structure of the non-abelian Born-Infeld lagrangian is completely known only up to $\mathcal{O}(\alpha'^2)$ terms [1, 3]. Results involving higher order corrections have only been partial in the sense that a non-abelian prescription that worked for $\mathcal{O}(\alpha'^2)$ terms [5] did not work at higher orders (see [6] and references therein) or that only some terms of a higher order correction in the effective lagrangian have been found [6], but not all.

So a good starting point to consider would be the determination of the complete $\mathcal{O}(\alpha'^3)$ terms in the non-abelian Born-Infeld lagrangian. This problem was studied a long time ago in [7] by means of the partial computation of the five-point amplitude in Open Superstring theory. Recent attempts have avoided this direct calculation, succeeding in arriving to an effective lagrangian [8, 9]. But controversy emerged since the $\mathcal{O}(\alpha'^3)$ terms of these effective lagrangians turned up to be non-equivalent [8, 10].

In this paper we reconsider the scattering amplitude approach to the effective lagrangian by calculating the complete open superstring five-point amplitude.

Our paper is organized as follows. In section 2 we give a brief review of N-gluon scattering amplitudes in the context of Open Superstring theory (at tree level). In section 3 we shortly give the known three and four-gluon amplitudes, as derived directly from the general N-gluon formula, presented in section 2. In section 4 we consider the low energy gluon lagrangian responsible for the previous three and four-gluon amplitudes (up to $\mathcal{O}(\alpha'^2)$ terms) and we introduce the $\mathcal{O}(\alpha'^3)$ lagrangian terms of [8]. In section 5 we develop the main result of this paper, namely, the five-gluon tree amplitude as derived from Open Superstring theory. In the final section, we make an analysis of the different existing versions of the lagrangian terms at order $\mathcal{O}(\alpha'^3)$, in light of our five-point amplitude result. Our main conclusion is that we find complete agreement with the one in [8]. Appendix A contains the kinematic factors which appear in the five-gluon tree amplitude. Appendix B contains the explicit expressions for the second and third order contributions, in α', to this amplitude. Appendices C and D constitute a fundamental support to all the effective lagrangian expressions appearing in this work since they contain the Feynman rules and the scattering amplitudes derived from these lagrangians, which match with all the corresponding scattering amplitudes derived from Open Superstring theory. We have treated these very involved calculations using the *Maple* version of the computer algebra package HIP [11].

While this paper was being written, the authors came across a recent preprint [12], in which the $\mathcal{O}(\alpha'^3)$ lagrangian terms of [8] were confirmed by studying α' deformations of $D = 10$ Super Yang-Mills theory up to order three, imposing supersymmetry order by order. Our work has been done with complete independence and without any reference to this preprint.

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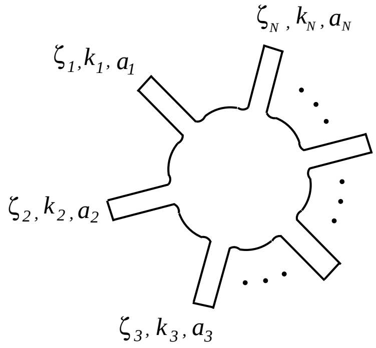

Figure 1: The planar open-string diagram.

2. Review of gluon tree amplitudes in Open Superstring theory

Upon quantization of the superstring it turns out that Lorentz invariance is violated unless $D = 10$ and non massive states ($M^2 = 0$) are part of the infinite spectrum [13, 14, 15]. These states describe a vector particle A^μ, and a spinor particle ψ^μ. If Chan-Paton factors are associated to the ends of the open superstring, then this allows for the introduction of a $\mathrm{U}(n)$ gauge group. In this case the couple (A_a^μ, ψ_a^μ) is the one describing $D = 10$ Supersymmetric Yang-Mills theory. Along this paper we will refer to the A_a^μ vector particle as a gluon. The tree level scattering amplitude of N gluons with polarizations $\zeta_1, \zeta_2, \ldots, \zeta_N$, external momenta $k_1, k_2, \ldots, k_N$[1] and colors $a_1, a_2, \ldots, a_N$, is calculated in Open Superstring theory as [13, 14, 15]

$$\mathcal{A}^{(N)} = i(2\pi)^{10}\delta^{10}(k_1 + k_2 + \cdots + k_N) \cdot \sum_{\mathrm{perm}'} \mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\ldots\lambda^{a_n})A(1, 2, \ldots, N)\,, \qquad (2.1)$$

where the sum $\sum_{\mathrm{perm}'}$ is over *all non-cyclic* permutations of the sets $\{\zeta_1, k_1, a_1\}$, $\{\zeta_2, k_2, a_2\}, \ldots, \{\zeta_N, k_N, a_N\}$, λ^{a_n} are the $\mathrm{U}(n)$ generators satisfying the relations (C.1) and the argument $1, 2, \ldots, N$ of the Lorentz factor $A(1, 2, \ldots, N)$ is a compact notation for ζ_1, $k_1; \zeta_2$, $k_2; \ldots; \zeta_N$, k_N. $A(1, 2, \ldots, N)$ corresponds to the N-particle scattering amplitude of open superstrings which do not carry color indices and which are placed among themselves in the specific ordering $\{1, 2, \ldots, N\}$ (modulo cyclic permutations). For example, the case of the ordering $\{1, 2, \ldots, N\}$ is shown in figure 1. The first step of the calculations presented in appendix D is to express all the amplitudes in the form given by eq. (2.1).

Using the vertex operator formalism, the following expression may be derived for the corresponding amplitude [13]:

$$\begin{aligned} A(1, 2, \ldots, N) = {} & 2\frac{g^{N-2}}{(2\alpha')^{7N/4+2}}(x_{N-1} - x_1)(x_N - x_1) \times \\ & \times \int dx_2 \ldots dx_{N-2} \int d\theta_1 \ldots d\theta_{N-2} \prod_{i>j}^{N} |x_i - x_j - \theta_i\theta_j|^{2\alpha' k_i \cdot k_j} \times \\ & \times \int d\phi_1 \ldots d\phi_N e^{f_N(\zeta, k, \theta, \phi)}\,, \end{aligned} \qquad (2.2)$$

[1]The momenta k_i are all assumed to be *inward* in the corresponding scattering process.

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where

$$f_N(\zeta,k,\theta,\phi)=\sum_{i\neq j}^{N}\frac{(\theta_i-\theta_j)\phi_i(\zeta_i\cdot k_j)(2\alpha')^{11/4}-1/2\phi_i\phi_j(\zeta_i\cdot\zeta_j)(2\alpha')^{9/2}}{x_i-x_j-\theta_i\theta_j}\,. \tag{2.3}$$

In (2.2) and (2.3) the x_i are all bosonic (commuting) variables, while the θ_i, together with the ϕ_i variables, are all fermionic (anticommuting) ones. There are some remarks about eqs. (2.2) and (2.3):

- Equation (2.2) contains all consistent factors that lead to the N-gluon tree amplitude in Yang-Mills theory, in the limit $\alpha'\to 0$.[2]

- The curious powers of $(2\alpha')$ that appear in eqs. (2.2) and (2.3) are just a consequence of a dimensional analysis of the corresponding formula in [13, section 7.3], in which the convention $\alpha'=1/2$ is used. As will be seen later, the α' expansion of the amplitudes contains only *integer* powers of it.

- The original amplitude $A(1,2,3,\ldots,N)$ is invariant under OSp(1|2) transformations of all the x_i and θ_i $(i=1,\ldots,N)$ [13]. As a consequence of this symmetry, the variables x_1, x_{N-1}, x_N, θ_{N-1} and θ_N, all behave like *free* parameters (the final result does not depend on them). Now, in eqs. (2.2) and (2.3), variables x_1, x_{N-1} and x_N have been kept arbitrary, while θ_{N-1} and θ_N have been set to 0.

- To guarantee the specific ordering of the superstrings, $\{1,2,3,\ldots,N\}$, the condition

$$x_1<x_2<x_3<\ldots<x_N \tag{2.4}$$

  should be imposed in eq. (2.2).

- Although not manifest, $A(1,2,3,\ldots,N)$ has the cyclic property.[3]

- The external momenta k_i and the polarizations ζ_i in (2.2) satisfy

$$\left.\begin{array}{ll} i)\ \text{Physical state condition:} & \zeta_1\cdot k_1=\zeta_2\cdot k_2=\ldots=\zeta_N\cdot k_N=0\\ ii)\ \text{On-shell condition:} & k_1^2=k_2^2=\ldots=k_n^2=0\end{array}\right\} \tag{2.5}$$

Our convention for the Minkowski metric is the following:

$$\eta_{\mu\nu}=\mathrm{diag}(-,+,+,\ldots,+)\,. \tag{2.6}$$

[2] In eq. (2.2) g is the Yang-Mills coupling constant.

[3] The formula in eq. (2.2) comes directly from the corresponding formula in [13, section 7.3], which does have the cyclic property.

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3. Review of three and four-gluon tree amplitudes

As an application of the formula in eq. (2.2) we have the well known three and four-point amplitudes. In the first case it becomes

$$
\begin{aligned}
A(1,2,3) = {} & 2\frac{g}{(2\alpha')^{29/4}}(x_2-x_1)(x_3-x_1)\times \\
& \times \int d\theta_1 |x_2-x_1-\theta_2\theta_1|^{2\alpha' k_2\cdot k_1}|x_3-x_1-\theta_3\theta_1|^{2\alpha' k_3\cdot k_1}|x_3-x_2-\theta_3\theta_2|^{2\alpha' k_3\cdot k_2}\times \\
& \times \int d\phi_1 d\phi_2 d\phi_3 e^{f_3(\zeta,k,\theta,\phi)}\,,
\end{aligned}
\tag{3.1}
$$

In this case there is no x integration (x_1, x_2 and x_3 are arbitrary, with the only condition $x_1 < x_2 < x_3$) and we have $\theta_2 = \theta_3 = 0$. Momentum conservation implies that $k_2 \cdot k_1 = k_3 \cdot k_1 = k_3 \cdot k_2 = 0$, so that $A(1,2,3)$ has a simple expression which, after substituted in $\mathcal{A}^{(3)}$ leads to [13, 15]

$$
\begin{aligned}
\mathcal{A}^{(3)} = {} & 2ig(2\pi)^{10}\delta^{10}(k_1+k_2+k_3)\times \\
& \times\left((\zeta_1\cdot k_2)(\zeta_2\cdot\zeta_3)+(\zeta_2\cdot k_3)(\zeta_3\cdot\zeta_1)+(\zeta_3\cdot k_1)(\zeta_1\cdot\zeta_2)\right)\mathrm{tr}(\lambda^{a_1}[\lambda^{a_2},\lambda^{a_3}])\,.
\end{aligned}
\tag{3.2}
$$

This is exactly the three-gluon tree amplitude, as derived from Yang-Mills theory (see eq. (D.2)): it has no superstring theory corrections. Now, for the four gluon tree amplitude, (2.2) becomes

$$
\begin{aligned}
A(1,2,3,4) = {} & 2\frac{g^2}{(2\alpha')^9}(x_3-x_1)(x_4-x_1)\times \\
& \times\int_{x_1}^{x_3} dx_2 \int d\theta_1 d\theta_2 |x_2-x_1-\theta_2\theta_1|^{2\alpha' k_2\cdot k_1}|x_3-x_1-\theta_3\theta_1|^{2\alpha' k_3\cdot k_1}|x_4-x_1-\theta_4\theta_1|^{2\alpha' k_4\cdot k_1}\times \\
& \times |x_3-x_2-\theta_3\theta_2|^{2\alpha' k_3\cdot k_2}|x_4-x_2-\theta_4\theta_2|^{2\alpha' k_4\cdot k_2}|x_4-x_3-\theta_4\theta_3|^{2\alpha' k_4\cdot k_3}\times \\
& \times\int d\phi_1 d\phi_2 d\phi_3 d\phi_4 e^{f_4(\zeta,k,\theta,\phi)}\,.
\end{aligned}
\tag{3.3}
$$

In this case the only x integration is carried over x_2 (where $x_1 < x_2 < x_3 < x_4$) and we also have $\theta_3 = \theta_4 = 0$. In order to simplify the calculations x_1, x_3 and x_4 are typically chosen to be 0, 1 and $+\infty$. After doing the calculations and introducing the Mandelstam variables,

$$
s = -2k_1\cdot k_2\,,\qquad t = -2k_1\cdot k_4\,,\qquad u = -2k_1\cdot k_3\,,
\tag{3.4}
$$

$A(1,2,3,4)$ can be obtained in a closed expression, which, after substituted in $\mathcal{A}^{(4)}$ leads to [13, 15]

$$
\begin{aligned}
\mathcal{A}^{(4)} = {} & 8ig^2(\alpha')^2(2\pi)^{10}\delta^{10}(k_1+k_2+k_3+k_4)\times \\
& \times\left[\frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}\{\mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4})+\mathrm{tr}(\lambda^{a_1}\lambda^{a_4}\lambda^{a_3}\lambda^{a_2})\}+\right. \\
& \quad +\frac{\Gamma(-\alpha' t)\Gamma(-\alpha' u)}{\Gamma(1-\alpha' t-\alpha' u)}\{\mathrm{tr}(\lambda^{a_1}\lambda^{a_4}\lambda^{a_2}\lambda^{a_3})+\mathrm{tr}(\lambda^{a_1}\lambda^{a_3}\lambda^{a_2}\lambda^{a_4})\}+
\end{aligned}
$$

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$$+\frac{\Gamma(-\alpha' u)\Gamma(-\alpha' s)}{\Gamma(1-\alpha' u-\alpha' s)}\{\mathrm{tr}(\lambda^{a_1}\lambda^{a_3}\lambda^{a_4}\lambda^{a_2})+\mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_4}\lambda^{a_3})\}\Bigg]+$$
$$\times K(\zeta_1,k_1;\zeta_2,k_2;\zeta_3,k_3;\zeta_4,k_4)\,, \tag{3.5}$$

where the kinematic factor $K(\zeta_1,k_1;\zeta_2,k_2;\zeta_3,k_3;\zeta_4,k_4)$ is given by

$$\begin{aligned}
K \;=\; & -\frac{1}{4}\Big[ts(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_4)+su(\zeta_2\cdot\zeta_3)(\zeta_1\cdot\zeta_4)+ut(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)\Big]+\\
& +\frac{1}{2}s\Big[(\zeta_1\cdot k_4)(\zeta_3\cdot k_2)(\zeta_2\cdot\zeta_4)+(\zeta_2\cdot k_3)(\zeta_4\cdot k_1)(\zeta_1\cdot\zeta_3)+\\
& \qquad +(\zeta_1\cdot k_3)(\zeta_4\cdot k_2)(\zeta_2\cdot\zeta_3)+(\zeta_2\cdot k_4)(\zeta_3\cdot k_1)(\zeta_1\cdot\zeta_4)\Big]+\\
& +\frac{1}{2}t\Big[(\zeta_2\cdot k_1)(\zeta_4\cdot k_3)(\zeta_3\cdot\zeta_1)+(\zeta_3\cdot k_4)(\zeta_1\cdot k_2)(\zeta_2\cdot\zeta_4)+\\
& \qquad +(\zeta_2\cdot k_4)(\zeta_1\cdot k_3)(\zeta_3\cdot\zeta_4)+(\zeta_3\cdot k_1)(\zeta_4\cdot k_2)(\zeta_2\cdot\zeta_1)\Big]+\\
& +\frac{1}{2}u\Big[(\zeta_1\cdot k_2)(\zeta_4\cdot k_3)(\zeta_3\cdot\zeta_2)+(\zeta_3\cdot k_4)(\zeta_2\cdot k_1)(\zeta_1\cdot\zeta_4)+\\
& \qquad +(\zeta_1\cdot k_4)(\zeta_2\cdot k_3)(\zeta_3\cdot\zeta_4)+(\zeta_3\cdot k_2)(\zeta_4\cdot k_1)(\zeta_1\cdot\zeta_2)\Big]
\end{aligned} \tag{3.6}$$

An important thing about $\mathcal{A}^{(4)}$, and that does not happen in $\mathcal{A}^{(3)}$, is the fact that it contains an infinite number of higher order corrections in α'. The coefficients of this α' expansion can all be determined in terms of the Riemann Zeta function, evaluated in integer values. For example, the first term involving Gamma functions in (3.5) has the following α' expansion:

$$\frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}=\frac{1}{\alpha'^2 st}-\frac{\pi^2}{6}-\zeta(3)(s+t)\alpha'+\mathcal{O}(\alpha'^2)\,. \tag{3.7}$$

After substituting this last expression and similar ones for the other terms involving Gamma functions in (3.5), the leading term (i.e., order zero in α') in $\mathcal{A}^{(4)}$ is nothing else than the Yang-Mills four-gluon tree amplitude; the first superstring theory contributions to $\mathcal{A}^{(4)}$ begin at order two in α' and they all have a common factor π^2; the next ones occur at order three in α' and they all have a common factor $\zeta(3)$, and so on.

4. Effective lagrangian up to $\mathcal{O}(\alpha'^2)$ and beyond

4.1 Effective lagrangian up to $\mathcal{O}(\alpha'^2)$

Knowledge of the on-shell scattering amplitudes of four gluons is enough to determine the effective lagrangian up to $\mathcal{O}(\alpha'^2)$. This has been known for a long time [1, 3] to be:

$$\begin{aligned}
\mathcal{L}_{(0,2)} \;=\; & -\frac{1}{4}tr\,(F_{\mu_1\mu_2}F^{\mu_1\mu_2})+\\
& +\left(2\pi\alpha'\right)^2 g^2\,\mathrm{tr}\bigg(\frac{1}{24}F_{\mu_1}^{\ \ \mu_2}F_{\mu_2}^{\ \ \mu_3}F_{\mu_3}^{\ \ \mu_4}F_{\mu_4}^{\ \ \mu_1}+\frac{1}{12}F_{\mu_1}^{\ \ \mu_2}F_{\mu_2}^{\ \ \mu_3}F_{\mu_4}^{\ \ \mu_1}F_{\mu_3}^{\ \ \mu_4}-\\
& \qquad -\frac{1}{48}F_{\mu_1}^{\ \ \mu_2}F_{\mu_2}^{\ \ \mu_1}F_{\mu_3}^{\ \ \mu_4}F_{\mu_4}^{\ \ \mu_3}-\frac{1}{96}F_{\mu_1}^{\ \ \mu_2}F_{\mu_3}^{\ \ \mu_4}F_{\mu_2}^{\ \ \mu_1}F_{\mu_4}^{\ \ \mu_3}\bigg)
\end{aligned} \tag{4.1}$$

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where the field strength is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\, g\, [A_\mu,\, A_\nu]\,. \tag{4.2}$$

Here, the index $(0,2)$ has been used to denote that this lagrangian contains terms of order 0 and 2 in α'. No order 1 terms in α' appear.

4.2 $\mathcal{O}(\alpha'^3)$ contributions to the effective lagrangian

As was argued in the previous section, the $\mathcal{A}^{(4)}$ amplitude receives, beyond $\mathcal{O}(\alpha'^2)$, $\mathcal{O}(\alpha'^3)$ and higher order contributions. In fact, using the α' expansion for the Gamma functions in $\mathcal{A}^{(4)}$ (see eq. (3.5)) some new $\mathcal{O}(\alpha'^3)$ terms may be derived in the effective lagrangian. These would be the D^2F^4 terms, as may be understood by dimensional analysis. Since $\mathcal{A}^{(3)}$ does not contain any α' terms at all, no D^4F^3 terms will be present in the effective lagrangian. So, the remaining $\mathcal{O}(\alpha'^3)$ terms can only be of F^5 type. To derive them it would be necessary to compute the five-gluon tree amplitude, $\mathcal{A}^{(5)}$, up to order three in α', which is going to be done in section 5.

At this point, it is important to say, as was already mentioned in the introduction, that in the literature there can be found three non-equivalent versions of the $\mathcal{O}(\alpha'^3)$ terms of the effective lagrangian (see references [8] and [10] for a detailed comparison of them). The first of them [7] was derived a long time ago by means of the computation of the five-gluon amplitude in Open Superstring theory. The other two versions are recent. One of them is derived from the effective action of $N=4$ super Yang-Mills in $D=4$ [9] and the other uses deformations of a particular kind of solutions of Yang-Mills theory [8] in any even dimensional spacetime. We will see, in the next sections, that our detailed calculation of the five gluon tree amplitude $\mathcal{A}^{(5)}$ matches completely the last of these three versions, due to Koerber and Sevrin. Their effective lagrangian, at order $\mathcal{O}(\alpha'^3)$, is the following [8]:[4]

$$\begin{aligned}
\mathcal{L}_{(3)} = &-(2\,\alpha')^3\, 2\,\zeta(3)\times \\
&\times \mathrm{tr}\Big[i\, g^3\, F_{\mu_1}^{\ \mu_2}\, F_{\mu_2}^{\ \mu_3}\, F_{\mu_3}^{\ \mu_4}\, F_{\mu_5}^{\ \mu_1}\, F_{\mu_4}^{\ \mu_5} + i\, g^3\, F_{\mu_1}^{\ \mu_2}\, F_{\mu_4}^{\ \mu_5}\, F_{\mu_2}^{\ \mu_3}\, F_{\mu_5}^{\ \mu_1}\, F_{\mu_3}^{\ \mu_4} - \\
&\quad - \frac{i\, g^3}{2}\, F_{\mu_1}^{\ \mu_2}\, F_{\mu_2}^{\ \mu_3}\, F_{\mu_4}^{\ \mu_5}\, F_{\mu_3}^{\ \mu_1}\, F_{\mu_5}^{\ \mu_4} - g^2\, F_{\mu_1}^{\ \mu_2} \left(D^{\mu_1} F_{\mu_3}^{\ \mu_4}\right) \left(D_{\mu_5} F_{\mu_2}^{\ \mu_3}\right) F_{\mu_4}^{\ \mu_5} + \\
&\quad + \frac{g^2}{2} \left(D^{\mu_1} F_{\mu_2}^{\ \mu_3}\right) \left(D_{\mu_1} F_{\mu_3}^{\ \mu_4}\right) F_{\mu_5}^{\ \mu_2}\, F_{\mu_4}^{\ \mu_5} + \frac{g^2}{2} \left(D^{\mu_1} F_{\mu_2}^{\ \mu_3}\right) F_{\mu_5}^{\ \mu_2} \left(D_{\mu_1} F_{\mu_3}^{\ \mu_4}\right) F_{\mu_4}^{\ \mu_5} - \\
&\quad - \frac{g^2}{8} \left(D^{\mu_1} F_{\mu_2}^{\ \mu_3}\right) F_{\mu_4}^{\ \mu_5} \left(D_{\mu_1} F_{\mu_3}^{\ \mu_2}\right) F_{\mu_5}^{\ \mu_4} + g^2 \left(D_{\mu_5} F_{\mu_1}^{\ \mu_2}\right) F_{\mu_3}^{\ \mu_4} \left(D^{\mu_1} F_{\mu_2}^{\ \mu_3}\right) F_{\mu_4}^{\ \mu_5} \Big]\,,
\end{aligned} \tag{4.3}$$

[4]There are two differences in the conventions used by the authors of [8] and ours:

i. They use anti-hermitian matrices as generators of the $\mathrm{U}(n)$ Lie algebra, while we use hermitian matrices. This implies that in our formulas in equations (4.1), (4.2) and (4.3), there are some signals and i factors that do not appear in their effective lagrangian.

ii. They introduce the coupling constant g only as a global factor $-1/g^2$ in the lagrangian, while we introduce it in the definition of $F_{\mu\nu}$, in eq. (4.2), and also as powers of it multiplying each individual term in the lagrangian.

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where, besides (4.2), the covariant derivative acting on some field ϕ is defined as

$$D_\mu \phi = \partial_\mu \phi - i\, g\, [A_\mu,\, \phi]\,. \tag{4.4}$$

5. Five-gluon tree amplitude in Open Superstring theory

In this section we develop the main result of this paper, namely, the five gluon tree amplitude, as derived from the Open Superstring theory. Our formula in eq. (2.2), in the case of five gluons becomes

$$\begin{aligned} A(1,2,3,4,5) \;=\;& 2\frac{g^3}{(2\alpha')^{43/4}}(x_4-x_1)(x_5-x_1)\times \\ &\times \int_{x_1}^{x_4} dx_3 \int_{x_1}^{x_3} dx_2 \int d\theta_1 d\theta_2 d\theta_3 \prod_{i>j}^{5} |x_i - x_j - \theta_i\theta_j|^{2\alpha' k_i\cdot k_j}\times \\ &\times \int d\phi_1 d\phi_2 d\phi_3 d\phi_4 d\phi_5 e^{f_5(\zeta,k,\theta,\phi)}\,, \end{aligned} \tag{5.1}$$

where $\theta_4 = \theta_5 = 0$. Starting from this formula and arriving to a final answer, where $A(1,2,3,4,5)$ is an explicit expression of α', the momenta k_i and the polarizations ζ_i, is a huge task. To save the reader from the very lengthy calculations involved in this task, we have decided to present the explicit expression for $A(1,2,3,4,5)$, up to $\mathcal{O}(\alpha'^3)$ terms, in subsection 5.1 and the details of the calculations in the following subsections.

5.1 Explicit expression for the 5-point amplitude

Up to $\mathcal{O}(\alpha'^3)$ terms, the final result for $A(1,2,3,4,5)$ has the following form:

$$A(1,2,3,4,5) = A^{(0)}(1,2,3,4,5) + A^{(2)}(1,2,3,4,5)\cdot\alpha'^2 + A^{(3)}(1,2,3,4,5)\cdot\alpha'^3 + \mathcal{O}(\alpha'^4)\,, \tag{5.2}$$

where the terms $A^{(0)}(1,2,3,4,5)$, $A^{(2)}(1,2,3,4,5)$ and $A^{(3)}(1,2,3,4,5)$ do not depend on α'. The first of these three terms is nothing else than the Yang-Mills five-gluon tree amplitude, whose expression is the following:[5]

$$\begin{aligned} &A^{(0)}(1,2,3,4,5) = \\ &2g^3\Bigg[(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)\times \\ &\quad\times\Bigg\{(\zeta_5\cdot k_1)\alpha_{23}\left\{\frac{1}{\rho\alpha_{12}}+\frac{1}{\rho\alpha_{23}}+\frac{1}{\alpha_{34}\phi}+\frac{1}{\alpha_{23}\phi}+\frac{1}{\alpha_{34}\alpha_{12}}\right\} - \\ &\qquad - (\zeta_5\cdot k_2)\alpha_{13}\left\{\frac{1}{\alpha_{34}\alpha_{12}}+\frac{1}{\rho\alpha_{12}}\right\} + \\ &\qquad + (\zeta_5\cdot k_3)\Bigg(\alpha_{24}\left\{\frac{1}{\alpha_{34}\alpha_{12}}+\frac{1}{\alpha_{34}\phi}\right\} + \end{aligned}$$

[5] The complete Yang-Mills five-gluon tree amplitude is constructed from (5.3) using the same rule given in eq. (2.1).

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$$
\begin{aligned}
&+\alpha_{23}\left\{\frac{1}{\rho\alpha_{12}}+\frac{1}{\rho\alpha_{23}}+\frac{1}{\alpha_{34}\phi}+\frac{1}{\alpha_{23}\phi}+\frac{1}{\alpha_{34}\alpha_{12}}\right\}\Bigg)\Bigg\}+\\
&+(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_4)\Bigg\{-(\zeta_5\cdot k_1)\alpha_{23}\left\{\frac{1}{\alpha_{23}\rho}+\frac{1}{\alpha_{23}\phi}\right\}+\\
&\qquad+(\zeta_5\cdot k_2)\left(\alpha_{34}\left\{\frac{1}{\alpha_{34}\phi}\right\}-\alpha_{23}\left\{\frac{1}{\alpha_{23}\rho}+\frac{1}{\alpha_{23}\phi}\right\}\right)-\\
&\qquad-(\zeta_5\cdot k_3)\alpha_{12}\left\{\frac{1}{\rho\alpha_{12}}\right\}\Bigg\}+\\
&+(\zeta_1\cdot\zeta_4)(\zeta_2\cdot\zeta_3)\Bigg\{(\zeta_5\cdot k_1)\left(\alpha_{34}\left\{\frac{1}{\alpha_{23}\phi}+\frac{1}{\alpha_{34}\phi}\right\}-\alpha_{13}\left\{\frac{1}{\alpha_{23}\rho}\right\}\right)-\\
&\qquad-(\zeta_5\cdot k_2)\frac{\alpha_{13}}{\alpha_{23}\rho}+(\zeta_5\cdot k_3)\alpha_{12}\left\{\frac{1}{\alpha_{23}\rho}+\frac{1}{\alpha_{12}\rho}\right\}\Bigg\}+\\
&+(\zeta_2\cdot\zeta_3)\Bigg\{(\zeta_5\cdot k_1)\Bigg((\zeta_1\cdot k_2)(\zeta_4\cdot k_3)\left\{\frac{1}{\rho\alpha_{12}}+\frac{1}{\rho\alpha_{23}}+\frac{1}{\alpha_{34}\phi}+\frac{1}{\alpha_{23}\phi}+\frac{1}{\alpha_{34}\alpha_{12}}\right\}-\\
&\qquad\quad-(\zeta_1\cdot k_3)(\zeta_4\cdot k_2)\left\{\frac{1}{\alpha_{23}\rho}+\frac{1}{\alpha_{23}\phi}\right\}\Bigg)+\\
&\qquad+(\zeta_5\cdot k_2)\Bigg((\zeta_1\cdot k_3)(\zeta_4\cdot k_1)\frac{1}{\alpha_{23}\rho}+\\
&\qquad\quad+(\zeta_1\cdot k_3)(\zeta_4\cdot k_3)\left\{\frac{1}{\alpha_{23}\alpha_{34}}+\frac{1}{\rho\alpha_{23}}-\frac{\alpha_{24}}{\alpha_{23}\phi\alpha_{34}}\right\}+\\
&\qquad\quad+(\zeta_1\cdot k_2)(\zeta_4\cdot k_3)\left\{\frac{1}{\rho\alpha_{12}}+\frac{1}{\rho\alpha_{23}}+\frac{1}{\alpha_{34}\phi}+\frac{1}{\alpha_{23}\phi}+\frac{1}{\alpha_{34}\alpha_{12}}\right\}+\\
&\qquad\quad+(\zeta_1\cdot k_4)(\zeta_4\cdot k_3)\left\{\frac{1}{\alpha_{23}\phi}+\frac{1}{\alpha_{34}\phi}\right\}\Bigg)-\\
&\qquad-(\zeta_5\cdot k_3)\Bigg((\zeta_1\cdot k_2)(\zeta_4\cdot k_1)\left\{\frac{1}{\alpha_{23}\rho}+\frac{1}{\alpha_{12}\rho}\right\}+\\
&\qquad\quad+(\zeta_1\cdot k_3)(\zeta_4\cdot k_2)\left\{\frac{1}{\alpha_{23}\rho}+\frac{1}{\alpha_{23}\phi}\right\}+\\
&\qquad\quad+(\zeta_1\cdot k_2)(\zeta_4\cdot k_2)\left\{\frac{1}{\alpha_{23}\alpha_{12}}+\frac{1}{\phi\alpha_{23}}-\frac{\alpha_{13}}{\alpha_{23}\rho\alpha_{12}}\right\}+\\
&\qquad\quad+(\zeta_1\cdot k_4)(\zeta_4\cdot k_2)\frac{1}{\alpha_{23}\phi}\Bigg)\Bigg\}+\\
&+(\zeta_1\cdot\zeta_4)\Bigg\{-(\zeta_5\cdot k_1)\Bigg((\zeta_2\cdot k_3)(\zeta_3\cdot k_4)\left\{\frac{1}{\alpha_{23}\phi}+\frac{1}{\alpha_{34}\phi}\right\}-(\zeta_2\cdot k_4)(\zeta_3\cdot k_2)\frac{1}{\alpha_{23}\phi}+\\
&\qquad\quad+(\zeta_2\cdot k_4)(\zeta_3\cdot k_4)\frac{1}{\alpha_{34}\phi}-(\zeta_2\cdot k_1)(\zeta_3\cdot k_4)\frac{1}{\alpha_{12}\alpha_{34}}\Bigg)+\\
&\qquad+(\zeta_5\cdot k_4)\Bigg((\zeta_2\cdot k_1)(\zeta_3\cdot k_2)\left\{\frac{1}{\alpha_{23}\rho}+\frac{1}{\alpha_{12}\rho}\right\}+(\zeta_2\cdot k_1)(\zeta_3\cdot k_1)\frac{1}{\alpha_{12}\rho}-\\
&\qquad\quad-(\zeta_2\cdot k_3)(\zeta_3\cdot k_1)\frac{1}{\alpha_{23}\rho}\Bigg)\Bigg\}+\\
&+(\text{cyclic permutations of indexes }(1,2,3,4,5))\Bigg],
\end{aligned}
\tag{5.3}
$$

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where $\alpha_{ij} = k_i \cdot k_j$, $\rho = \alpha_{12} + \alpha_{13} + \alpha_{23}$ and $\phi = \alpha_{23} + \alpha_{24} + \alpha_{34}$. $A^{(2)}(1,2,3,4,5)$ and $A^{(3)}(1,2,3,4,5)$ have similar expressions to the one shown in (5.3) for $A^{(0)}(1,2,3,4,5)$. Their explicit expressions are given in appendix B. As expected, in (5.2) there is no linear term in α'.

5.2 Fixing some of the free parameters in $A(1,2,3,4,5)$

In this subsection we begin the derivation of each of the terms in formula eq. (5.2). As a starting point we consider (5.1). Our only aim in this subsection is to simplify a little this last formula, after fixing $\theta_4 = 0$, $\theta_5 = 0$, and $x_5 = +\infty$ (x_1 and x_4 will be fixed to 0 and 1, respectively, in subsection 5.4). For this purpose it is necessary to expand the $exp(f_5(\zeta, k, \theta, \phi))$ term in eq. (5.1) and to do some of the Grassmann integrations. To start with, in eq. (2.3) we have that

$$f_5(\zeta, k, \theta, \phi) = \sum_{i \neq j}^{5} \frac{(\theta_i - \theta_j)\phi_i(\zeta_i \cdot k_j)(2\alpha')^{11/4} - 1/2\phi_i\phi_j(\zeta_i \cdot \zeta_j)(2\alpha')^{9/2}}{x_i - x_j - \theta_i\theta_j}\,, \tag{5.4}$$

so $f_5(\zeta, k, \theta, \phi)$ may be split into two sums, which can be expanded in powers of $1/x_5$ as follows:

$$\sum_{i \neq j}^{5} \frac{(\theta_i - \theta_j)\phi_i(\zeta_i \cdot k_j)}{x_i - x_j - \theta_i\theta_j} = A + \frac{B}{x_5} + \mathcal{O}\left(\frac{1}{{x_5}^2}\right) \tag{5.5}$$

and

$$\frac{1}{2}\sum_{i \neq j}^{5} \frac{\phi_i\phi_j(\zeta_i \cdot \zeta_j)}{x_i - x_j - \theta_i\theta_j} = C + \frac{D}{x_5} + \mathcal{O}\left(\frac{1}{{x_5}^2}\right). \tag{5.6}$$

After fixing $\theta_4 = 0$ and $\theta_5 = 0$, the bosonic terms A, B, C and D in this last formula are given by

$$\begin{aligned}
A = &\left[\theta_1\left(\frac{\zeta_1 \cdot k_2}{x_1 - x_2} + \frac{\zeta_1 \cdot k_3}{x_1 - x_3} + \frac{\zeta_1 \cdot k_4}{x_1 - x_4}\right) - \theta_2\frac{\zeta_1 \cdot k_2}{x_1 - x_2} - \theta_3\frac{\zeta_1 \cdot k_3}{x_1 - x_3}\right]\phi_1 + \\
&+ \left[-\theta_1\frac{\zeta_2 \cdot k_1}{x_2 - x_1} + \theta_2\left(\frac{\zeta_2 \cdot k_1}{x_2 - x_1} + \frac{\zeta_2 \cdot k_3}{x_2 - x_3} + \frac{\zeta_2 \cdot k_4}{x_2 - x_4}\right) - \theta_3\frac{\zeta_2 \cdot k_3}{x_2 - x_3}\right]\phi_2 + \\
&+ \left[-\theta_1\frac{\zeta_3 \cdot k_1}{x_3 - x_1} - \theta_2\frac{\zeta_3 \cdot k_2}{x_3 - x_2} + \theta_3\left(\frac{\zeta_3 \cdot k_1}{x_3 - x_1} + \frac{\zeta_3 \cdot k_2}{x_3 - x_2} + \frac{\zeta_3 \cdot k_4}{x_3 - x_4}\right)\right]\phi_3 - \\
&- \left[\theta_1\frac{\zeta_4 \cdot k_1}{x_4 - x_1} + \theta_2\frac{\zeta_4 \cdot k_2}{x_4 - x_2} + \theta_3\frac{\zeta_4 \cdot k_3}{x_4 - x_3}\right]\phi_4\,,
\end{aligned} \tag{5.7}$$

$$\begin{aligned}
B = &-\theta_1(\zeta_1 \cdot k_5)\phi_1 - \theta_2(\zeta_2 \cdot k_5)\phi_2 - \theta_3(\zeta_3 \cdot k_5)\phi_3 - \\
&- \Big[\theta_1(\zeta_5 \cdot k_1) + \theta_2(\zeta_5 \cdot k_2) + \theta_3(\zeta_5 \cdot k_3)\Big]\phi_5\,,
\end{aligned} \tag{5.8}$$

$$\begin{aligned}
C = &\frac{\phi_2\phi_1(\zeta_2 \cdot \zeta_1)}{x_2 - x_1 - \theta_2\theta_1} + \frac{\phi_3\phi_1(\zeta_3 \cdot \zeta_1)}{x_3 - x_1 - \theta_3\theta_1} + \frac{\phi_4\phi_1(\zeta_4 \cdot \zeta_1)}{x_4 - x_1} + \frac{\phi_3\phi_2(\zeta_3 \cdot \zeta_2)}{x_3 - x_2 - \theta_3\theta_2} + \\
&+ \frac{\phi_4\phi_2(\zeta_4 \cdot \zeta_2)}{x_4 - x_2} + \frac{\phi_4\phi_3(\zeta_4 \cdot \zeta_3)}{x_4 - x_3},
\end{aligned} \tag{5.9}$$

$$D = \phi_5\phi_1(\zeta_5 \cdot \zeta_1) + \phi_5\phi_2(\zeta_5 \cdot \zeta_2) + \phi_5\phi_3(\zeta_5 \cdot \zeta_3) + \phi_5\phi_4(\zeta_5 \cdot \zeta_4)\,. \tag{5.10}$$

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Now, multiplying separately the factors containing x_5 in the $\prod$ term of (5.1), using the on-shell condition (2.5) and using the sums given in eqs. (5.5) and (5.6) for the $f_5(\zeta,k,\theta,\phi)$ term in the exponential, we have that

$$\begin{aligned}
A(1,2,3,4,5) &= 2g^3(2\alpha')(x_4-x_1)(x_5-x_1)\times \qquad (5.11)\\
&\times \int_{x_1}^{x_4} dx_3 \int_{x_1}^{x_3} dx_2 \int d\theta_1 d\theta_2 d\theta_3 \prod_{i>j}^{4} |x_i - x_j - \theta_i\theta_j|^{2\alpha' k_i\cdot k_j} \times\\
&\times \left[1-(2\alpha')\Big((k_1\cdot k_5)x_1+(k_2\cdot k_5)x_2+(k_3\cdot k_5)x_3+(k_4\cdot k_5)x_4\Big)\frac{1}{x_5}+\mathcal{O}\left(\frac{1}{x_5^2}\right)\right]\times\\
&\times \int d\phi_1 d\phi_2 d\phi_3 d\phi_4 d\phi_5 \left\{\left[\frac{AC^2}{2}+(2\alpha')\frac{A^3}{6}\right]+\right.\\
&\qquad\left.+\left[\frac{2ACD+BC^2}{2}+(2\alpha')\frac{3A^2BC+A^3D}{6}\right]\frac{1}{x_5}+\mathcal{O}\left(\frac{1}{x_5^2}\right)\right\}.
\end{aligned}$$

In this last formula, the term $[\frac{AC^2}{2}+(2\alpha')\frac{A^3}{6}]$ gives no contribution after the Grassmann integration in $d\phi_5$ is done, since neither A nor C contain the ϕ_5 variable. So, after choosing $x_5\to+\infty$, eq. (5.11) becomes

$$\begin{aligned}
A(1,2,3,4,5) &= 2g^3(2\alpha')(x_4-x_1)\times\\
&\times \int_{x_1}^{x_4} dx_3 \int_{x_1}^{x_3} dx_2 \int d\theta_1 d\theta_2 d\theta_3 \prod_{i>j}^{4} |x_i - x_j - \theta_i\theta_j|^{2\alpha' k_i\cdot k_j} \times\\
&\times \int d\phi_1 d\phi_2 d\phi_3 d\phi_4 d\phi_5 \left\{\frac{2ACD+BC^2}{2}+(2\alpha')\frac{3A^2BC+A^3D}{6}\right\}. \qquad (5.12)
\end{aligned}$$

Although θ_4 appears in the $\prod$ term of this formula, it is assumed to be 0. So the only free parameters in eq. (5.12) are x_1 and x_4 (satisfying $x_4 > x_1$).

5.3 Calculating the ϕ Grassmann integration

In this subsection we integrate all the ϕ_i Grassmann variables appearing in (5.12).

i. As a first step we substitute the expressions of A, B, C and D, given respectively in (5.7), (5.8), (5.9) and (5.10), in the first term of the ϕ_i Grassmann integration in formula (5.12). After some steps we arrive to

$$\begin{aligned}
\int d\phi_1 d\phi_2 d\phi_3 d\phi_4 d\phi_5 \left\{\frac{2ACD+BC^2}{2}\right\} &= -S_{12345}\theta_1 + S_{21345}\theta_2 + S_{32145}\theta_3\\
&\quad -(T_{12345}+T_{21345}+T_{32145})\theta_1\theta_2\theta_3\,, \qquad (5.13)
\end{aligned}$$

where

$$\begin{aligned}
&S_{12345} = \qquad (5.14)\\
&-\left(\frac{\zeta_1\cdot k_2}{x_2-x_1}+\frac{\zeta_1\cdot k_3}{x_3-x_1}+\frac{\zeta_1\cdot k_4}{x_4-x_1}\right)\times\\
&\times\left(\frac{(\zeta_3\cdot\zeta_2)(\zeta_5\cdot\zeta_4)}{x_3-x_2}-\frac{(\zeta_4\cdot\zeta_2)(\zeta_5\cdot\zeta_3)}{x_4-x_2}+\frac{(\zeta_4\cdot\zeta_3)(\zeta_5\cdot\zeta_2)}{x_4-x_3}\right)-
\end{aligned}$$

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$$
\begin{aligned}
&-\frac{\zeta_2\cdot k_1}{x_2-x_1}\left(-\frac{(\zeta_3\cdot\zeta_1)(\zeta_5\cdot\zeta_4)}{x_3-x_1}+\frac{(\zeta_4\cdot\zeta_1)(\zeta_5\cdot\zeta_3)}{x_4-x_1}-\frac{(\zeta_4\cdot\zeta_3)(\zeta_5\cdot\zeta_1)}{x_4-x_3}\right)-\\
&-\frac{\zeta_3\cdot k_1}{x_3-x_1}\left(\frac{(\zeta_2\cdot\zeta_1)(\zeta_5\cdot\zeta_4)}{x_2-x_1}-\frac{(\zeta_4\cdot\zeta_1)(\zeta_5\cdot\zeta_2)}{x_4-x_1}+\frac{(\zeta_4\cdot\zeta_2)(\zeta_5\cdot\zeta_1)}{x_4-x_2}\right)-\\
&-\frac{\zeta_4\cdot k_1}{x_4-x_1}\left(-\frac{(\zeta_2\cdot\zeta_1)(\zeta_5\cdot\zeta_3)}{x_2-x_1}+\frac{(\zeta_3\cdot\zeta_1)(\zeta_5\cdot\zeta_2)}{x_3-x_1}-\frac{(\zeta_3\cdot\zeta_2)(\zeta_5\cdot\zeta_1)}{x_3-x_2}\right)+\\
&+(\zeta_5\cdot k_1)\left(\frac{(\zeta_2\cdot\zeta_1)(\zeta_4\cdot\zeta_3)}{(x_2-x_1)(x_4-x_3)}-\frac{(\zeta_3\cdot\zeta_1)(\zeta_4\cdot\zeta_2)}{(x_3-x_1)(x_4-x_2)}+\frac{(\zeta_4\cdot\zeta_1)(\zeta_3\cdot\zeta_2)}{(x_4-x_1)(x_3-x_2)}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
T_{12345} &= \frac{(\zeta_3\cdot\zeta_2)(\zeta_5\cdot\zeta_4)}{(x_3-x_2)^2}\left(\frac{\zeta_1\cdot k_2}{x_2-x_1}+\frac{\zeta_1\cdot k_3}{x_3-x_1}+\frac{\zeta_1\cdot k_4}{x_4-x_1}\right)-\\
&\quad-\frac{(\zeta_3\cdot\zeta_2)(\zeta_5\cdot\zeta_1)}{(x_3-x_2)^2}\frac{\zeta_4\cdot k_1}{x_4-x_1}-\frac{(\zeta_4\cdot\zeta_1)(\zeta_3\cdot\zeta_2)}{(x_3-x_2)^2}\frac{\zeta_5\cdot k_1}{x_4-x_1}\,. \qquad (5.15)
\end{aligned}
$$

The coefficients S_{21345}, S_{32145}, T_{21345} and T_{32145} appearing in (5.13) are to be derived using the corresponding expressions for S_{12345} and T_{12345} and interchanging the corresponding indexes.

ii. As a second step we now substitute the expressions of A, B, C and D in the second term of the ϕ_i Grassmann integration in (5.12). Again, after some steps we arrive to

$$
\begin{aligned}
&\int d\phi_1 d\phi_2 d\phi_3 d\phi_4 d\phi_5\left\{\frac{3A^2BC+A^3D}{6}\right\}= \qquad (5.16)\\
&\quad-\Bigg[U_{12345}\frac{\zeta_2\cdot\zeta_1}{x_2-x_1}+U_{13245}\frac{\zeta_3\cdot\zeta_1}{x_3-x_1}+U_{23145}\frac{\zeta_3\cdot\zeta_2}{x_3-x_2}+\\
&\qquad+V_{12345}\frac{\zeta_4\cdot\zeta_1}{x_4-x_1}+V_{21345}\frac{\zeta_4\cdot\zeta_2}{x_4-x_2}+V_{32145}\frac{\zeta_4\cdot\zeta_3}{x_4-x_3}+\\
&\qquad+W_{1234}(\zeta_5\cdot\zeta_1)+W_{2134}(\zeta_5\cdot\zeta_2)+W_{3124}(\zeta_5\cdot\zeta_3)+Z_{1234}(\zeta_5\cdot\zeta_4)\Bigg]\theta_1\theta_2\theta_3\,,
\end{aligned}
$$

where

$$
\begin{aligned}
U_{12345} &= -\left[\frac{\zeta_3\cdot k_2}{x_3-x_2}\cdot\frac{\zeta_4\cdot k_3}{x_4-x_3}+\left(\frac{\zeta_3\cdot k_2}{x_3-x_2}+\frac{\zeta_3\cdot k_1}{x_3-x_1}+\frac{\zeta_3\cdot k_4}{x_3-x_4}\right)\frac{\zeta_4\cdot k_2}{x_4-x_2}\right](\zeta_5\cdot k_1)+\\
&\quad+\left[\frac{\zeta_3\cdot k_1}{x_3-x_1}\cdot\frac{\zeta_4\cdot k_3}{x_4-x_3}+\left(\frac{\zeta_3\cdot k_2}{x_3-x_2}+\frac{\zeta_3\cdot k_1}{x_3-x_1}+\frac{\zeta_3\cdot k_4}{x_3-x_4}\right)\frac{\zeta_4\cdot k_1}{x_4-x_1}\right](\zeta_5\cdot k_2)-\\
&\quad-\left[\frac{\zeta_3\cdot k_1}{x_3-x_1}\cdot\frac{\zeta_4\cdot k_2}{x_4-x_2}-\frac{\zeta_3\cdot k_2}{x_3-x_2}\cdot\frac{\zeta_4\cdot k_1}{x_4-x_1}\right](\zeta_5\cdot k_3), \qquad (5.17)\\
V_{12345} &= -\Bigg[\left(\frac{\zeta_2\cdot k_1}{x_2-x_1}+\frac{\zeta_2\cdot k_3}{x_2-x_3}+\frac{\zeta_2\cdot k_4}{x_2-x_4}\right)\left(\frac{\zeta_3\cdot k_2}{x_3-x_2}+\frac{\zeta_3\cdot k_1}{x_3-x_1}+\frac{\zeta_3\cdot k_4}{x_3-x_4}\right)-\\
&\quad-\frac{\zeta_2\cdot k_3}{x_2-x_3}\cdot\frac{\zeta_3\cdot k_2}{x_3-x_2}\Bigg](\zeta_5\cdot k_1)- \qquad (5.18)\\
&\quad-\left[\left(\frac{\zeta_3\cdot k_2}{x_3-x_2}+\frac{\zeta_3\cdot k_1}{x_3-x_1}+\frac{\zeta_3\cdot k_4}{x_3-x_4}\right)\frac{\zeta_2\cdot k_1}{x_2-x_1}+\frac{\zeta_2\cdot k_3}{x_2-x_3}\cdot\frac{\zeta_3\cdot k_1}{x_3-x_1}\right](\zeta_5\cdot k_2)-
\end{aligned}
$$

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$$
-\left[\frac{\zeta_2\cdot k_1}{x_2-x_1}\cdot\frac{\zeta_3\cdot k_2}{x_3-x_2}+\left(\frac{\zeta_2\cdot k_1}{x_2-x_1}+\frac{\zeta_2\cdot k_3}{x_2-x_3}+\frac{\zeta_2\cdot k_4}{x_2-x_4}\right)\frac{\zeta_3\cdot k_1}{x_3-x_1}\right](\zeta_5\cdot k_3)\,,
$$

$$
\begin{aligned}
W_{1234} = & \left[\frac{\zeta_3\cdot k_2}{x_3-x_2}\cdot\frac{\zeta_4\cdot k_3}{x_4-x_3}+\left(\frac{\zeta_3\cdot k_2}{x_3-x_2}+\frac{\zeta_3\cdot k_1}{x_3-x_1}+\frac{\zeta_3\cdot k_4}{x_3-x_4}\right)\frac{\zeta_4\cdot k_2}{x_4-x_2}\right]\frac{\zeta_2\cdot k_1}{x_2-x_1}+ \\
& +\left[\frac{\zeta_3\cdot k_1}{x_3-x_1}\cdot\frac{\zeta_4\cdot k_3}{x_4-x_3}-\left(\frac{\zeta_3\cdot k_2}{x_3-x_2}+\frac{\zeta_3\cdot k_1}{x_3-x_1}+\frac{\zeta_3\cdot k_4}{x_3-x_4}\right)\frac{\zeta_4\cdot k_1}{x_4-x_1}\right]\times \\
& \times\left(\frac{\zeta_2\cdot k_1}{x_2-x_1}+\frac{\zeta_2\cdot k_3}{x_2-x_3}+\frac{\zeta_2\cdot k_4}{x_2-x_4}\right)- \\
& -\left[\frac{\zeta_3\cdot k_1}{x_3-x_1}\cdot\frac{\zeta_4\cdot k_2}{x_4-x_2}-\frac{\zeta_3\cdot k_2}{x_3-x_2}\cdot\frac{\zeta_4\cdot k_1}{x_4-x_1}\right]\frac{\zeta_2\cdot k_3}{x_2-x_3}
\end{aligned} \tag{5.19}
$$

and

$$
\begin{aligned}
Z_{1234} = & \left[\left(\frac{\zeta_2\cdot k_1}{x_2-x_1}+\frac{\zeta_2\cdot k_3}{x_2-x_3}+\frac{\zeta_2\cdot k_4}{x_2-x_4}\right)\left(\frac{\zeta_3\cdot k_2}{x_3-x_2}+\frac{\zeta_3\cdot k_1}{x_3-x_1}+\frac{\zeta_3\cdot k_4}{x_3-x_4}\right)-\right. \\
& \left.-\frac{\zeta_2\cdot k_3}{x_2-x_3}\cdot\frac{\zeta_3\cdot k_2}{x_3-x_2}\right]\left(\frac{\zeta_1\cdot k_2}{x_1-x_2}+\frac{\zeta_1\cdot k_3}{x_1-x_3}+\frac{\zeta_1\cdot k_4}{x_1-x_4}\right)- \\
& -\left[\left(\frac{\zeta_3\cdot k_2}{x_3-x_2}+\frac{\zeta_3\cdot k_1}{x_3-x_1}+\frac{\zeta_3\cdot k_4}{x_3-x_4}\right)\frac{\zeta_2\cdot k_1}{x_2-x_1}+\frac{\zeta_2\cdot k_3}{x_2-x_3}\cdot\frac{\zeta_3\cdot k_1}{x_3-x_1}\right]\frac{\zeta_1\cdot k_2}{x_1-x_2}- \\
& -\left[\frac{\zeta_2\cdot k_1}{x_2-x_1}\cdot\frac{\zeta_3\cdot k_2}{x_3-x_2}-\left(\frac{\zeta_2\cdot k_1}{x_2-x_1}+\frac{\zeta_2\cdot k_3}{x_2-x_3}+\frac{\zeta_2\cdot k_4}{x_2-x_4}\right)\frac{\zeta_3\cdot k_1}{x_3-x_1}\right]\frac{\zeta_1\cdot k_3}{x_1-x_3}\,.
\end{aligned} \tag{5.20}
$$

iii. We summarize the results of this subsection by substituting eqs. (5.13) and (5.16) in eq. (5.12). So the expression for $A(1,2,3,4,5)$ turns into

$$
\begin{aligned}
& A(1,2,3,4,5) = \\
& 2g^3(2\alpha')(x_4-x_1)\int_{x_1}^{x_4}dx_3\int_{x_1}^{x_3}dx_2\int d\theta_1 d\theta_2 d\theta_3\prod_{i>j}^{4}|x_i-x_j-\theta_i\theta_j|^{2\alpha' k_i\cdot k_j}\times \\
& \times\left[\Big\{-S_{12345}\theta_1+S_{21345}\theta_2+S_{32145}\theta_3-(T_{12345}+T_{21345}+T_{32145})\theta_1\theta_2\theta_3\Big\}-\right. \\
& \quad -(2\alpha')\Big\{U_{12345}\frac{\zeta_2\cdot\zeta_1}{x_2-x_1}+U_{13245}\frac{\zeta_3\cdot\zeta_1}{x_3-x_1}+U_{23145}\frac{\zeta_3\cdot\zeta_2}{x_3-x_2}+V_{12345}\frac{\zeta_4\cdot\zeta_1}{x_4-x_1}+ \\
& \qquad +V_{21345}\frac{\zeta_4\cdot\zeta_2}{x_4-x_2}+V_{32145}\frac{\zeta_4\cdot\zeta_3}{x_4-x_3}+W_{1234}(\zeta_5\cdot\zeta_1)+W_{2134}(\zeta_5\cdot\zeta_2)+ \\
& \qquad \left.+W_{3124}(\zeta_5\cdot\zeta_3)+Z_{1234}(\zeta_5\cdot\zeta_4)\Big\}\theta_1\theta_2\theta_3\right],
\end{aligned} \tag{5.21}
$$

where all the S, T, U, V, W and Z coefficients are bosonic and known (they depend on x_i, k_i and ζ_i).

5.4 Calculating the θ Grassmann integration and deriving the final expression for $A(1,2,3,4,5)$

In this subsection we integrate the θ_i Grassmann variables and we then fix x_1 and x_4, arriving to the final expression for $A(1,2,3,4,5)$. As an intermediate step, to clear out any confusion with the lengthy expression involved, we consider some specific terms of the scattering amplitude, showing how their coefficients are obtained.

i. We start expanding the $\prod$ factor appearing in (5.21) as

$$\prod_{i>j}^{4} |x_i - x_j - \theta_i\theta_j|^{2\alpha' k_i\cdot k_j} = \tag{5.22}$$
$$\prod_{i>j}^{4}(x_i - x_j)^{2\alpha' k_i\cdot k_j}\left[1 - 2\alpha'\left\{\frac{(k_2\cdot k_1)\theta_2\theta_1}{x_2 - x_1} + \frac{(k_3\cdot k_2)\theta_3\theta_2}{x_3 - x_2} + \frac{(k_3\cdot k_1)\theta_3\theta_1}{x_3 - x_1}\right\}\right].$$

After substituting (5.22) in (5.21) and then integrating over θ_1, θ_2 and θ_3, we have that

$$A(1,2,3,4,5) = \tag{5.23}$$
$$2g^3(2\alpha')(x_4 - x_1)\int_{x_1}^{x_4} dx_3 \int_{x_1}^{x_3} dx_2 \prod_{i>j}^{4}(x_i - x_j)^{2\alpha' k_i\cdot k_j} \times$$
$$\times\Bigg[\Bigg\{T_{12345} + T_{21345} + T_{32145} +$$
$$+ (2\alpha')\left(\frac{(k_3\cdot k_2)S_{12345}}{x_3 - x_2} + \frac{(k_3\cdot k_1)S_{21345}}{x_3 - x_1} - \frac{(k_2\cdot k_1)S_{32145}}{x_2 - x_1}\right)\Bigg\} +$$
$$+ (2\alpha')\Bigg\{U_{12345}\frac{\zeta_2\cdot\zeta_1}{x_2 - x_1} + U_{13245}\frac{\zeta_3\cdot\zeta_1}{x_3 - x_1} + U_{23145}\frac{\zeta_3\cdot\zeta_2}{x_3 - x_2} +$$
$$+ V_{12345}\frac{\zeta_4\cdot\zeta_1}{x_4 - x_1} + V_{21345}\frac{\zeta_4\cdot\zeta_2}{x_4 - x_2} + V_{32145}\frac{\zeta_4\cdot\zeta_3}{x_4 - x_3}$$
$$+ W_{1234}(\zeta_5\cdot\zeta_1) + W_{2134}(\zeta_5\cdot\zeta_2) + W_{3124}(\zeta_5\cdot\zeta_3) + Z_{1234}(\zeta_5\cdot\zeta_4)\Bigg\}\Bigg].$$

ii. Now, using the expressions for the S, T, U, V, W and Z known coefficients, given in the previous subsection, we can see in (5.23) that

- The first curly bracket is the one responsible for terms of the type

$$(\zeta_i\cdot\zeta_j)(\zeta_k\cdot\zeta_l)(\zeta_m\cdot k_n)\times\{\text{kinematic factor}\}\,, \tag{5.24}$$

in the scattering amplitude.

- The second curly bracket is the one responsible for the remaining terms of the scattering amplitude, which are of the type

$$(\zeta_i\cdot\zeta_j)(\zeta_k\cdot k_l)(\zeta_m\cdot k_n)(\zeta_p\cdot k_q)\times\{\text{kinematic factor}\}\,. \tag{5.25}$$

We will denote the resulting integrals of each curly bracket as $I_{(\zeta\cdot\zeta)^2(\zeta\cdot k)}$ and $I_{(\zeta\cdot\zeta)(\zeta\cdot k)^3}$, respectively. As an example, we will look for the scattering amplitude terms which contribute as $(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)(\zeta_5\cdot k_i)$. They will be found only in the first curly bracket, as part of the $I_{(\zeta\cdot\zeta)^2(\zeta\cdot k)}$ integral. It turns out that these terms only appear in four of the six S and T coefficients, in the following way

$$S_{12345} \to +\frac{(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)}{(x_2 - x_1)(x_4 - x_3)}\cdot(\zeta_5\cdot k_1)$$

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$$\begin{aligned}
S_{21345} &\to -\frac{(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)}{(x_2-x_1)(x_4-x_3)}\cdot(\zeta_5\cdot k_2)\\
S_{32145} &\to -\frac{(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)}{(x_2-x_1)(x_4-x_3)}\cdot(\zeta_5\cdot k_3)\\
T_{32145} &\to -\frac{(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)}{(x_2-x_1)^2}\cdot\frac{(\zeta_5\cdot k_3)}{x_4-x_3}\,.
\end{aligned}\tag{5.26}$$

So their contribution to $A(1,2,3,4,5)$ is given by the integral

$$\begin{aligned}
&I_{(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)(\zeta_5\cdot k_i)}=\\
&\quad 2g^3(2\alpha')(x_4-x_1)\int_{x_1}^{x_4}dx_3\int_{x_1}^{x_3}dx_2\prod_{i>j}^{4}(x_i-x_j)^{2\alpha' k_i\cdot k_j}\frac{(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)}{(x_2-x_1)(x_4-x_3)}\times\\
&\quad\times\left\{(2\alpha')(k_2\cdot k_3)\frac{(\zeta_5\cdot k_1)}{x_3-x_2}-(2\alpha')(k_3\cdot k_1)\frac{(\zeta_5\cdot k_2)}{x_3-x_1}+(2\alpha')(k_1\cdot k_2)\frac{(\zeta_5\cdot k_3)}{x_2-x_1}\right.\\
&\qquad\left.-\frac{(\zeta_5\cdot k_3)}{x_2-x_1}\right\},
\end{aligned}\tag{5.27}$$

which can then be written as

$$\begin{aligned}
I_{(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)(\zeta_5\cdot k_i)} = {}& 2g^3(2\alpha')(x_4-x_1)(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)\times\\
&\times\Big[2\alpha' L_2\cdot(k_2\cdot k_3)(\zeta_5\cdot k_1)-2\alpha' L_3\cdot(k_1\cdot k_3)(\zeta_5\cdot k_2)+\\
&\quad+(2\alpha'(k_1\cdot k_2)-1)L_5\cdot(\zeta_5\cdot k_3)\Big]\,.
\end{aligned}\tag{5.28}$$

Here, L_2, L_3 and L_5 are part of a list of known kinematic factors (double integrals which depend in α' and the momenta k_i) given in subsection A.1 of appendix A.[6] We now reconsider eq. (5.23) and fix there $x_1=0$ and $x_4=1$. Carrying over there the same procedure already done in this subsection for terms of the type $(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)(\zeta_5\cdot k_i)\times\{\text{kinematic factor}\}$, but for every term of the type (5.24) and every term of the type (5.25), we finally arrive to five-point amplitude:

$$\begin{aligned}
&A(1,2,3,4,5)=\\
&2g^3(2\alpha')^2\Big[(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)\Big\{(\zeta_5\cdot k_1)(k_2\cdot k_3)L_2-(\zeta_5\cdot k_2)(k_1\cdot k_3)L_3+\\
&\qquad\qquad+(\zeta_5\cdot k_3)\left((k_2\cdot k_4){L_3}'+(k_2\cdot k_3)L_2\right)\Big\}+\\
&\quad+(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_4)\Big\{-(\zeta_5\cdot k_1)(k_2\cdot k_3)L_7+\\
&\qquad\qquad+(\zeta_5\cdot k_2)\left((k_3\cdot k_4){L_1}'-(k_2\cdot k_3)L_7\right)-\\
&\qquad\qquad-(\zeta_5\cdot k_3)(k_1\cdot k_2)L_1\Big\}+\\
&\quad+(\zeta_1\cdot\zeta_4)(\zeta_2\cdot\zeta_3)\Big\{(\zeta_5\cdot k_1)\left((k_3\cdot k_4){K_4}'-(k_1\cdot k_3)K_5\right)-
\end{aligned}$$

[6] In subsection A.1, x_1 and x_4 have already been fixed to 0 and 1, respectively.

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$$
\begin{aligned}
&- (\zeta_5\cdot k_2)(k_1\cdot k_3)K_5 + (\zeta_5\cdot k_3)(k_1\cdot k_2)K_4\Big\} + \\
&+ (\zeta_2\cdot\zeta_3)\Big\{(\zeta_5\cdot k_1)\left(((\zeta_1\cdot k_2)(\zeta_4\cdot k_3)L_2 - (\zeta_1\cdot k_3)(\zeta_4\cdot k_2)L_7\right) + \\
&\quad + (\zeta_5\cdot k_2)\Big((\zeta_1\cdot k_3)(\zeta_4\cdot k_1)K_5 + (\zeta_1\cdot k_3)(\zeta_4\cdot k_3){L_4}' + \\
&\quad\quad + (\zeta_1\cdot k_2)(\zeta_4\cdot k_3)L_2 + (\zeta_1\cdot k_4)(\zeta_4\cdot k_3){K_4}'\Big) - \\
&\quad - (\zeta_5\cdot k_3)\Big((\zeta_1\cdot k_2)(\zeta_4\cdot k_1)K_4 + (\zeta_1\cdot k_3)(\zeta_4\cdot k_2)L_7 + \\
&\quad\quad + (\zeta_1\cdot k_2)(\zeta_4\cdot k_2)L_4 + (\zeta_1\cdot k_4)(\zeta_4\cdot k_2){K_5}'\Big)\Big\} + \\
&+ (\zeta_1\cdot\zeta_4)\Big\{(\zeta_5\cdot k_2)\left(((\zeta_2\cdot k_1)(\zeta_3\cdot k_4)K_2 - (\zeta_2\cdot k_4)(\zeta_3\cdot k_1)K_3\right) - \\
&\quad - (\zeta_5\cdot k_1)\Big((\zeta_2\cdot k_3)(\zeta_3\cdot k_4){K_4}' - (\zeta_2\cdot k_4)(\zeta_3\cdot k_2){K_5}' + \\
&\quad\quad + (\zeta_2\cdot k_4)(\zeta_3\cdot k_4){K_1}' - (\zeta_2\cdot k_1)(\zeta_3\cdot k_4)K_2\Big) + \\
&+ (\zeta_5\cdot k_4)\Big((\zeta_2\cdot k_1)(\zeta_3\cdot k_2)K_4 + (\zeta_2\cdot k_1)(\zeta_3\cdot k_1)K_1 - \\
&\quad - (\zeta_2\cdot k_3)(\zeta_3\cdot k_1)K_5 - (\zeta_2\cdot k_4)(\zeta_3\cdot k_1)K_3\Big)\Big\} + \\
&+ (\text{cyclic permutations of indexes } (1,2,3,4,5))\Big]\,. \qquad (5.29)
\end{aligned}
$$

The K_i, ${K_i}'$, L_i and ${L_i}'$ kinematic factors in this formula are all detailed in subsection A.1 of appendix A. Once again, they depend on α' and the momenta k_i (they do not depend on the polarizations ζ_i).

In deriving (5.29) from (5.23), we have used the cyclic property satisfied by the open superstring tree level amplitudes, mentioned in section 2.

The complete on-shell five gluon tree amplitude, $\mathcal{A}^{(5)}$, would be obtained from eq. (5.29) by using the rule given in eq. (2.1).

Expression (5.29) is exact in the sense that there is no α' expansion in it: besides the $(2\alpha')^2$ factor in the beginning, the rest of the α' dependence comes all in the kinematic factors. These factors may be expanded in an α' series to any desired order. In appendix A, an α' expansion of these factors is done up to linear terms in α', which is enough information to derive, afterwards, the tree level scattering amplitude up to $\mathcal{O}(\alpha'^3)$ terms, as presented in subsection 5.1, in eq. (5.2).

At a first look to eqs. (5.28) and (5.29) there may seem to be some trouble when comparing the $(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)(\zeta_5\cdot k_i)$ terms of each, because for them to be equal it would be necessary to have

$$\left(2\alpha'(k_1\cdot k_2) - 1\right)L_5 = 2\alpha'\left((k_2\cdot k_4){L_3}' + (k_2\cdot k_3)L_2\right). \qquad (5.30)$$

This last formula is valid indeed and it is part of a set of relations among the kinematic factors contained in subsection A.2 of appendix A.

6. Final remarks and conclusions

In this paper we have derived the open superstring five-point amplitude of massless bosons

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(eq. (5.29)). We have also derived its α' expansion up to $\mathcal{O}(\alpha'^3)$ terms (eq. (5.2)), finding explicit expressions for the first three non zero contributions to this expansion: $A^{(0)}(1,2,3,4,5)$, $A^{(2)}(1,2,3,4,5)$ and $A^{(3)}(1,2,3,4,5)$[7] (given respectively in eqs. (5.3), (B.1) and (B.2)). As far as we know this is the first time that the on-shell five-point amplitude has been completely calculated up to order three in α'. We have used the different order contributions of the five-point amplitude to analyze the gluon effective lagrangian up to $\mathcal{O}(\alpha'^3)$ terms. The $A^{(0)}(1,2,3,4,5)$ and $A^{(2)}(1,2,3,4,5)$ contributions confirm the gluon effective lagrangian up to $\mathcal{O}(\alpha'^2)$ terms (given in eq. (4.1)): this has been considered in appendix D (see the final paragraph).

The $\mathcal{O}(\alpha'^3)$ contribution to the effective lagrangian, $\mathcal{L}_{(3)}$, deserves some special attention, since three non-equivalent versions of it have been published in the literature ([7, 9] and [8]).[8] The three versions agree in the D^2F^4 terms and only differ in the F^5 terms [8, 10]. Now, the D^2F^4 terms can be directly derived from the four-point amplitude $\mathcal{A}^{(4)}$ expanded up to $\mathcal{O}(\alpha'^3)$ terms [6] (see eqs. (3.5) and (3.7)). In fact, we have confirmed in appendix D that the D^2F^4 terms of $\mathcal{L}_{(3)}$, given in eq. (4.3), give a four-point amplitude that agrees with the corresponding one obtained in Open Superstring theory (see eqs. (D.10) and (D.11)). So, from a scattering amplitude approach, the real importance in the computation of a five-point amplitude lies in the determination of the F^5 terms. $A^{(3)}(1,2,3,4,5)$ turns out to be of crucial importance in this last purpose. Using its expression we have found complete agreement with the five-point amplitude coming from the $\mathcal{L}_{(3)}$ lagrangian in eq. (4.3), due to the authors of [8] (see appendix D). It should be mentioned that this lagrangian had also passed a test done in [10] by the same authors.

So, our main conclusion agrees completely with the one in [8], namely, that the bosonic terms of the non-abelian Born-Infeld supersymmetric lagrangian, up to $\mathcal{O}(\alpha'^3)$, are given by[9]

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4} tr\left(F_{\mu_1\mu_2}F^{\mu_1\mu_2}\right) + \\
& + \left(2\pi\alpha'\right)^2 g^2 \,\mathrm{tr}\Big[\frac{1}{24} F_{\mu_1}^{\ \mu_2} F_{\mu_2}^{\ \mu_3} F_{\mu_3}^{\ \mu_4} F_{\mu_4}^{\ \mu_1} + \frac{1}{12} F_{\mu_1}^{\ \mu_2} F_{\mu_2}^{\ \mu_3} F_{\mu_4}^{\ \mu_1} F_{\mu_3}^{\ \mu_4} - \\
& \qquad - \frac{1}{48} F_{\mu_1}^{\ \mu_2} F_{\mu_2}^{\ \mu_1} F_{\mu_3}^{\ \mu_4} F_{\mu_4}^{\ \mu_3} - \frac{1}{96} F_{\mu_1}^{\ \mu_2} F_{\mu_3}^{\ \mu_4} F_{\mu_2}^{\ \mu_1} F_{\mu_4}^{\ \mu_3}\Big] - \\
& - (2\,\alpha')^3\, 2\,\zeta(3)\,\mathrm{tr}\Big[\, i\,g^3 F_{\mu_1}^{\ \mu_2} F_{\mu_2}^{\ \mu_3} F_{\mu_3}^{\ \mu_4} F_{\mu_5}^{\ \mu_1} F_{\mu_4}^{\ \mu_5} + i\,g^3 F_{\mu_1}^{\ \mu_2} F_{\mu_4}^{\ \mu_5} F_{\mu_2}^{\ \mu_3} F_{\mu_5}^{\ \mu_1} F_{\mu_3}^{\ \mu_4} - \\
& \qquad - \frac{i\,g^3}{2} F_{\mu_1}^{\ \mu_2} F_{\mu_2}^{\ \mu_3} F_{\mu_4}^{\ \mu_5} F_{\mu_3}^{\ \mu_1} F_{\mu_5}^{\ \mu_4} - \\
& \qquad - g^2 F_{\mu_1}^{\ \mu_2} \left(D^{\mu_1} F_{\mu_3}^{\ \mu_4}\right)\left(D_{\mu_5} F_{\mu_2}^{\ \mu_3}\right) F_{\mu_4}^{\ \mu_5} +
\end{aligned}
$$

[7]The $A^{(1)}(1,2,3,4,5)$ term, that would give the order one contribution in α', turns out to be exactly zero.

[8]In [8, 10] a detailed comparison among the three versions has been done, concluding their non equivalence.

[9]There is a misprint in equation (5.1) of [10], which contains the complete effective lagrangian up to $\mathcal{O}(\alpha'^3)$ corrections: some terms have an additional factor 1/4.

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$$
\begin{aligned}
&+\frac{g^2}{2}\left(D^{\mu_1} F_{\mu_2}{}^{\mu_3}\right)\left(D_{\mu_1} F_{\mu_3}{}^{\mu_4}\right) F_{\mu_5}{}^{\mu_2} F_{\mu_4}{}^{\mu_5}+\\
&+\frac{g^2}{2}\left(D^{\mu_1} F_{\mu_2}{}^{\mu_3}\right) F_{\mu_5}{}^{\mu_2}\left(D_{\mu_1} F_{\mu_3}{}^{\mu_4}\right) F_{\mu_4}{}^{\mu_5}-\\
&-\frac{g^2}{8}\left(D^{\mu_1} F_{\mu_2}{}^{\mu_3}\right) F_{\mu_4}{}^{\mu_5}\left(D_{\mu_1} F_{\mu_3}{}^{\mu_2}\right) F_{\mu_5}{}^{\mu_4}+\\
&+g^2\left(D_{\mu_5} F_{\mu_1}{}^{\mu_2}\right) F_{\mu_3}{}^{\mu_4}\left(D^{\mu_1} F_{\mu_2}{}^{\mu_3}\right) F_{\mu_4}{}^{\mu_5}\Bigg] .
\end{aligned}
\tag{6.1}
$$

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A. Kinematic factors

A.1 List of kinematic factors

The K_i, $K_i{}'$, L_i and $L_i{}'$ kinematic factors, present in eq. (5.29), are all defined as double integrals of the form

$$
\int_0^1 dx_3 \int_0^{x_3} dx_2 x_3^{2\alpha'\alpha_{13}}(1-x_3)^{2\alpha'\alpha_{34}} x_2{}^{2\alpha'\alpha_{12}}(1-x_2)^{2\alpha'\alpha_{24}}(x_3-x_2)^{2\alpha'\alpha_{23}} \cdot \left\{ \begin{matrix} \kappa(x_2,x_3) \\ \lambda(x_2,x_3) \end{matrix} \right\},
\tag{A.1}
$$

where

$$
\alpha_{ij} = k_i \cdot k_j
\tag{A.2}
$$

and where the specific functions $\kappa(x_2, x_3)$ and $\lambda(x_2, x_3)$, associated to each of the kinematic factors, are given in table 1

$\kappa(x_2,x_3)$	Kinematic Factor	$\lambda(x_2,x_3)$	Kinematic Factor
$1/\{x_2 \cdot x_3\}$	K_1	$1/\{x_2 \cdot (1-x_2) \cdot x_3\}$	L_1
$1/\{(1-x_2) \cdot (1-x_3)\}$	$K_1{}'$	$1/\{(1-x_2) \cdot x_3 \cdot (1-x_3)\}$	$L_1{}'$
$1/\{x_2 \cdot (1-x_3)\}$	K_2	$1/\{x_2 \cdot (1-x_3) \cdot (x_3-x_2)\}$	L_2
$1/\{(1-x_2) \cdot x_3\}$	K_3	$1/\{x_2 \cdot x_3 \cdot (1-x_3)\}$	L_3
$1/\{x_2 \cdot (x_3-x_2)\}$	K_4	$1/\{x_2 \cdot (1-x_2) \cdot (1-x_3)\}$	$L_3{}'$
$1/\{(1-x_3) \cdot (x_3-x_2)\}$	$K_4{}'$	$1/\{x_2 \cdot (1-x_2) \cdot (x_3-x_2)\}$	L_4
$1/\{x_3 \cdot (x_3-x_2)\}$	K_5	$1/\{x_3 \cdot (1-x_3) \cdot (x_3-x_2)\}$	$L_4{}'$
$1/\{(1-x_2) \cdot (x_3-x_2)\}$	$K_5{}'$	$1/\{x_2{}^2 \cdot (1-x_3)\}$	L_5
$1/\{(x_3-x_2)^2\}$	K_6	$1/\{(1-x_2) \cdot x_3{}^2\}$	L_6
		$1/\{(1-x_2) \cdot x_3 \cdot (x_3-x_2)\}$	L_7

Table 1: List of kinematic factors.

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A.2 Relations among the kinematic factors

These integrals are not all independent. There is a *duality* operation $\star$ that interchanges some of the α_{ij} among themselves as follows,

$$\alpha_{12} \leftrightarrow \alpha_{34}, \alpha_{13} \leftrightarrow \alpha_{24} \qquad \text{and} \quad \alpha_{23} \leftrightarrow \alpha_{23}\,, \tag{A.3}$$

and such that every *primed* kinematic factor can be obtained from the corresponding *non primed* one by means of the *duality* operation:

$$K_i{}' = \star\{K_i\}\,, \qquad L_i{}' = \star\{L_i\}\,. \tag{A.4}$$

The kinematic factors K_2, K_3, K_6, L_2 and L_7 are self-dual:

$$K_2 = \star\{K_2\}\,, \qquad K_3 = \star\{K_3\}\,, \qquad K_6 = \star\{K_6\}\,, \qquad L_2 = \star\{L_2\}\,, \qquad L_7 = \star\{L_7\}\,. \tag{A.5}$$

The formulas in eqs. (A.4) and (A.5) can easily be proved by first changing the order of integration in the double integrals and then doing the substitution $x_2 \to 1 - x_3{}'$, $x_3 \to 1 - x_2{}'$. There are also some additional relations which can be proved using integration by parts:

$$\begin{aligned}
\alpha_{34}K_2 &= \alpha_{13}K_1 + \alpha_{23}K_4 \\
\alpha_{24}K_3 &= \alpha_{12}K_1 - \alpha_{23}K_5 \\
(2\alpha'\alpha_{23} - 1)K_6 &= 2\alpha' \left(\alpha_{34}K_4{}' - \alpha_{13}K_5\right) \\
\alpha_{13}L_1 &= \alpha_{34}L_3{}' - \alpha_{23}L_4{}' \\
(2\alpha'\alpha_{12} - 1)L_5 &= 2\alpha' \left(\alpha_{24}L_3{}' + \alpha_{23}L_2\right) \\
(2\alpha'\alpha_{13} - 1)L_6 &= 2\alpha' \left(\alpha_{34}L_1{}' - \alpha_{23}L_7\right)\,.
\end{aligned} \tag{A.6}$$

A.3 Explicit expression for the kinematic factors up to $\mathcal{O}(\alpha')$ terms

A matter of quite interest is to have an α' series for all these kinematic factors, since this allows to have the superstring corrections to the Yang-Mills five-gluon tree amplitude, $\mathcal{A}^{(5)}$, at different orders in α'. At least this is desired to be done up to $\mathcal{O}(\alpha')$ terms, since this is enough to obtain the effective lagrangian up to $\mathcal{O}(\alpha'^3)$. When looking for the α' expansion of the kinematic factors, some care must be taken with some of the double integrals, because they need to be regularized. In the list given in subsection A.1 this is necessary to be done with K_6 and L_5. The third and fifth relations in (A.6) assume that this regularization has been taken into account. After this regularization has been done, any of the kinematic factors listed in the tables may be calculated in terms of Beta functions and the Hypergeometric function ${}_3F_2$, as was done in [7]. From this kind of result, in principle an α' series can be obtained. We prefer, instead, to do a formal α' expansion for each kinematic factor in a different way. As an example we will show our procedure for

$$L_1 = \int_0^1 dx_3 x_3^{2\alpha'\alpha_{13}-1}(1-x_3)^{2\alpha'\alpha_{34}} \int_0^{x_3} dx_2 {x_2}^{2\alpha'\alpha_{12}-1}(1-x_2)^{2\alpha'\alpha_{24}-1}(x_3-x_2)^{2\alpha'\alpha_{23}} \tag{A.7}$$

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We start making the substitution $x_2 = u \cdot x_3$ in the inner integral (so x_3 acts as a constant in it), leading to

$$L_1 = \int_0^1 dx_3 x_3^{2\alpha'\rho-1}(1-x_3)^{2\alpha'\alpha_{34}} \int_0^1 du u^{2\alpha'\alpha_{12}-1}(1-u)^{2\alpha'\alpha_{23}}(1-ux_3)^{2\alpha'\alpha_{24}-1}, \quad \text{(A.8)}$$

where

$$\rho = \alpha_{12} + \alpha_{13} + \alpha_{23}. \quad \text{(A.9)}$$

Now, in (A.8) L_1 may be written as

$$\begin{aligned} L_1 = \;& B(2\alpha'\rho, 2\alpha'\alpha_{34}+1) \cdot B(2\alpha'\alpha_{12}, 2\alpha'\alpha_{23}+1) + \\ &+ \int_0^1 dx_3 x_3^{2\alpha'\rho-1} \underbrace{(1-x_3)^{2\alpha'\alpha_{34}} \int_0^1 du u^{2\alpha'\alpha_{12}-1}(1-u)^{2\alpha'\alpha_{23}}}_{I} \left\{(1-ux_3)^{2\alpha'\alpha_{24}-1} - 1\right\} \end{aligned} \quad \text{(A.10)}$$

(where $B(\lambda, \mu)$ is the Euler Beta function), such that the double integral I does have a well defined power series in α'. Since the α' expansion of the Beta functions that appear in (A.10) is known, the only thing that is missing is the series in α' for the integral term, which can be written as

$$\begin{aligned} I = & \left\{\int_0^1 dx_3 \int_0^1 \frac{du}{1-ux_3}\right\} + \\ &+ (2\alpha')\Bigg[\rho\left\{\int_0^1 dx_3 ln(x_3) \int_0^1 du \frac{1}{1-ux_3}\right\} + \alpha_{34}\left\{\int_0^1 dx_3 ln(1-x_3) \int_0^1 du \frac{1}{1-ux_3}\right\} + \\ &\quad + \alpha_{12}\left\{\int_0^1 dx_3 \int_0^1 du \frac{ln(u)}{1-ux_3}\right\} + \alpha_{23}\left\{\int_0^1 dx_3 \int_0^1 du \frac{ln(1-u)}{u(1-ux_3)}\right\} + \\ &\quad + \alpha_{24}\left\{\int_0^1 \frac{dx_3}{x_3} \int_0^1 du \frac{ln(1-ux_3)}{u(1-ux_3)}\right\}\Bigg] + \mathcal{O}(\alpha'^2). \end{aligned} \quad \text{(A.11)}$$

Each of the double integrals that appear in (A.11) can be calculated in terms of $\zeta(2) = \pi^2/6$ and $\zeta(3)$, where $\zeta(z)$ is the Riemann Zeta function.[10] The result is the following:

$$I = \frac{\pi^2}{6} + 2\alpha'\left[-\zeta(3)\rho - 2\zeta(3)\alpha_{34} - \zeta(3)\alpha_{12} - 2\zeta(3)\alpha_{23} - 2\zeta(3)\alpha_{24}\right] + \mathcal{O}(\alpha'^2). \quad \text{(A.12)}$$

Now, using the α' expansion for I and the one for the Beta functions appearing in (A.10), in that equation we finally have that

$$\begin{aligned} L_1 = & \frac{1}{(2\alpha')^2}\left\{\frac{1}{\rho \cdot \alpha_{12}}\right\} + \frac{\pi^2}{6}\left\{1 - \frac{\alpha_{34}}{\alpha_{12}} - \frac{\alpha_{23}}{\rho}\right\} + \\ &+ 2\zeta(3)\alpha'\left\{-\rho - 2\alpha_{34} - \alpha_{12} - 2\alpha_{23} - 2\alpha_{24} + \frac{\alpha_{34}^2}{\alpha_{12}} + \frac{\alpha_{23}^2}{\rho} + \frac{\alpha_{23} \cdot \alpha_{12}}{\rho} + \frac{\rho \cdot \alpha_{34}}{\alpha_{12}}\right\} + \mathcal{O}(\alpha'^2). \end{aligned}$$

[10] The interested reader can find a very nice method to calculate all these integrals, and more complicated ones, with the aid of *Harmonic Polylogarithms* [16] and the *harmpol* package, that uses computer language FORM [17].

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Doing analogous procedures to this one and using the relations among the kinematic factors, given in subsection A.2, we obtain the following list:

$$
\begin{aligned}
K_1 =& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\rho\cdot\alpha_{12}}\right\} - \frac{\pi^2}{6}\left\{\frac{\alpha_{34}}{\alpha_{12}}+\frac{\alpha_{23}}{\rho}\right\} + \\
&+ 2\zeta(3)\alpha'\left\{-\alpha_{24}+\frac{\alpha_{34}\cdot\rho}{\alpha_{12}}+\frac{\alpha_{12}\cdot\alpha_{23}}{\rho}+\frac{\alpha_{34}^2}{\alpha_{12}}+\frac{\alpha_{23}^2}{\rho}\right\} + \mathcal{O}(\alpha'^2)\,, \qquad (A.13)\\
K_1{}' =& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\phi\cdot\alpha_{34}}\right\} - \frac{\pi^2}{6}\left\{\frac{\alpha_{12}}{\alpha_{34}}+\frac{\alpha_{23}}{\phi}\right\} + \\
&+ 2\zeta(3)\alpha'\left\{-\alpha_{13}+\frac{\alpha_{12}\cdot\phi}{\alpha_{34}}+\frac{\alpha_{34}\cdot\alpha_{23}}{\phi}+\frac{\alpha_{12}^2}{\alpha_{34}}+\frac{\alpha_{23}^2}{\phi}\right\} + \mathcal{O}(\alpha'^2)\,, \qquad (A.14)\\
K_2 =& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\alpha_{12}\cdot\alpha_{34}}\right\} - \frac{\pi^2}{6}\left\{\frac{\rho}{\alpha_{12}}+\frac{\alpha_{23}}{\alpha_{34}}+\frac{\alpha_{24}}{\alpha_{34}}\right\} + \\
&+ 2\zeta(3)\alpha'\left\{2\alpha_{24}+\frac{\alpha_{34}\cdot\rho}{\alpha_{12}}+\frac{\rho^2}{\alpha_{12}}+\frac{\alpha_{23}^2}{\alpha_{34}}+\frac{\alpha_{24}^2}{\alpha_{34}}+2\frac{\alpha_{23}\cdot\alpha_{24}}{\alpha_{34}}+\frac{\alpha_{12}\cdot\alpha_{23}}{\alpha_{34}}+\frac{\alpha_{12}\cdot\alpha_{24}}{\alpha_{34}}\right\} + \\
&+ \mathcal{O}(\alpha'^2)\,, \qquad (A.15)\\
K_3 =& \frac{\pi^2}{6} - 2\zeta(3)\alpha'\left\{\rho+2\alpha_{23}+2\alpha_{34}+\alpha_{12}+\alpha_{24}\right\} + \mathcal{O}(\alpha'^2)\,, \qquad (A.16)\\
K_4 =& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\rho\cdot\alpha_{12}}+\frac{1}{\rho\cdot\alpha_{23}}\right\} - \frac{\pi^2}{6}\left\{\frac{\alpha_{24}}{\alpha_{23}}+\frac{\alpha_{34}}{\alpha_{23}}+\frac{\alpha_{12}}{\rho}+\frac{\alpha_{23}}{\rho}+\frac{\alpha_{34}}{\alpha_{12}}\right\} + \\
&+ 2\zeta(3)\alpha'\left\{\alpha_{24}+\frac{\alpha_{24}^2}{\alpha_{23}}+2\frac{\alpha_{34}\cdot\alpha_{24}}{\alpha_{23}}+\frac{\alpha_{34}\cdot\rho}{\alpha_{12}}+2\frac{\alpha_{12}\cdot\alpha_{23}}{\rho}+\frac{\alpha_{34}^2}{\alpha_{23}}+\frac{\alpha_{12}^2}{\rho}+\frac{\rho\cdot\alpha_{24}}{\alpha_{23}}+\right.\\
&\qquad\left.+\frac{\rho\cdot\alpha_{34}}{\alpha_{23}}+\frac{\alpha_{23}^2}{\rho}+\frac{\alpha_{34}^2}{\alpha_{12}}\right\} + \mathcal{O}(\alpha'^2)\,, \qquad (A.17)\\
K_4{}' =& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\phi\cdot\alpha_{34}}+\frac{1}{\phi\cdot\alpha_{23}}\right\} - \frac{\pi^2}{6}\left\{\frac{\alpha_{13}}{\alpha_{23}}+\frac{\alpha_{12}}{\alpha_{23}}+\frac{\alpha_{34}}{\phi}+\frac{\alpha_{23}}{\phi}+\frac{\alpha_{12}}{\alpha_{34}}\right\} + \\
&+ 2\zeta(3)\alpha'\left\{\alpha_{13}+\frac{\alpha_{13}^2}{\alpha_{23}}+2\frac{\alpha_{12}\cdot\alpha_{13}}{\alpha_{23}}+\frac{\alpha_{12}\cdot\phi}{\alpha_{34}}+2\frac{\alpha_{34}\cdot\alpha_{23}}{\phi}+\frac{\alpha_{12}^2}{\alpha_{23}}+\frac{\alpha_{34}^2}{\phi}+\frac{\phi\cdot\alpha_{13}}{\alpha_{23}}+\right.\\
&\qquad\left.+\frac{\phi\cdot\alpha_{12}}{\alpha_{23}}+\frac{\alpha_{23}^2}{\phi}+\frac{\alpha_{12}^2}{\alpha_{34}}\right\} + \mathcal{O}(\alpha'^2)\,, \qquad (A.18)\\
K_5 =& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\alpha_{23}\cdot\rho}\right\} - \frac{\pi^2}{6}\left\{\frac{\alpha_{34}}{\alpha_{23}}+\frac{\alpha_{12}}{\rho}+\frac{\alpha_{24}}{\alpha_{23}}\right\} + \\
&+ 2\zeta(3)\alpha'\left\{2\alpha_{24}+\frac{\alpha_{12}^2}{\rho}+\frac{\alpha_{34}^2}{\alpha_{23}}+\frac{\rho\cdot\alpha_{34}}{\alpha_{23}}+\frac{\rho\cdot\alpha_{24}}{\alpha_{23}}+2\frac{\alpha_{34}\cdot\alpha_{24}}{\alpha_{23}}+\frac{\alpha_{12}\cdot\alpha_{23}}{\rho}+\frac{\alpha_{24}^2}{\alpha_{23}}\right\} + \\
&+ \mathcal{O}(\alpha'^2)\,, \qquad (A.19)\\
K_5{}' =& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\alpha_{23}\cdot\phi}\right\} - \frac{\pi^2}{6}\left\{\frac{\alpha_{12}}{\alpha_{23}}+\frac{\alpha_{34}}{\phi}+\frac{\alpha_{13}}{\alpha_{23}}\right\} + \\
&+ 2\zeta(3)\alpha'\left\{2\alpha_{13}+\frac{\alpha_{34}^2}{\phi}+\frac{\alpha_{12}^2}{\alpha_{23}}+\frac{\phi\cdot\alpha_{12}}{\alpha_{23}}+\frac{\phi\cdot\alpha_{13}}{\alpha_{23}}+2\frac{\alpha_{12}\cdot\alpha_{13}}{\alpha_{23}}+\frac{\alpha_{34}\cdot\alpha_{23}}{\phi}+\frac{\alpha_{13}^2}{\alpha_{23}}\right\} + \\
&+ \mathcal{O}(\alpha'^2)\,, \qquad (A.20)\\
L_1 =& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\rho\cdot\alpha_{12}}\right\} + \frac{\pi^2}{6}\left\{1-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{23}}{\rho}\right\} +
\end{aligned}
$$

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$$
\begin{aligned}
&+2\zeta(3)\alpha'\left\{-\rho-2\alpha_{34}-\alpha_{12}-2\alpha_{23}-2\alpha_{24}+\frac{\alpha_{34}^2}{\alpha_{12}}+\frac{\alpha_{23}^2}{\rho}+\frac{\alpha_{23}\cdot\alpha_{12}}{\rho}+\frac{\rho\cdot\alpha_{34}}{\alpha_{12}}\right\}+\\
&+\mathcal{O}(\alpha'^2)\,, \qquad \text{(A.21)}
\end{aligned}
$$

$$
\begin{aligned}
{L_1}' =\;& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\phi\cdot\alpha_{34}}\right\}+\frac{\pi^2}{6}\left\{1-\frac{\alpha_{12}}{\alpha_{34}}-\frac{\alpha_{23}}{\phi}\right\}+\\
&+2\zeta(3)\alpha'\left\{-\phi-2\alpha_{12}-\alpha_{34}-2\alpha_{23}-2\alpha_{13}+\frac{\alpha_{12}^2}{\alpha_{34}}+\frac{\alpha_{23}^2}{\phi}+\frac{\alpha_{23}\cdot\alpha_{34}}{\phi}+\frac{\phi\cdot\alpha_{12}}{\alpha_{34}}\right\}+\\
&+\mathcal{O}(\alpha'^2)\,, \qquad \text{(A.22)}
\end{aligned}
$$

$$
\begin{aligned}
L_2 =\;& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\rho\cdot\alpha_{12}}+\frac{1}{\phi\cdot\alpha_{34}}+\frac{1}{\rho\cdot\alpha_{23}}+\frac{1}{\phi\cdot\alpha_{23}}+\frac{1}{\alpha_{12}\cdot\alpha_{34}}\right\}+\frac{\pi^2}{6}\left\{1-\frac{\alpha_{12}}{\rho}-\frac{\alpha_{34}}{\phi}-\right.\\
&\left.-\frac{\alpha_{12}}{\alpha_{34}}-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\rho}{\alpha_{12}}-\frac{\alpha_{23}}{\phi}-\frac{\rho}{\alpha_{23}}-\frac{\alpha_{23}}{\rho}-\frac{\alpha_{23}}{\alpha_{34}}-\frac{\alpha_{24}}{\alpha_{34}}-\frac{\alpha_{24}}{\alpha_{23}}\right\}+\\
&+2\zeta(3)\alpha'\left\{-\phi+3\alpha_{24}+\frac{\alpha_{12}^2}{\alpha_{34}}+\frac{\alpha_{23}^2}{\rho}+\frac{\rho^2}{\alpha_{12}}+\frac{\alpha_{34}^2}{\alpha_{12}}+\frac{\alpha_{34}^2}{\alpha_{23}}+\frac{\alpha_{34}^2}{\phi}+\frac{\alpha_{12}^2}{\rho}+\frac{\alpha_{23}^2}{\alpha_{34}}+\right.\\
&\qquad+\frac{\alpha_{23}^2}{\phi}+\frac{\alpha_{24}^2}{\alpha_{34}}+\frac{\alpha_{24}^2}{\alpha_{23}}+\frac{\rho^2}{\alpha_{23}}+2\frac{\rho\cdot\alpha_{34}}{\alpha_{23}}+2\frac{\rho\cdot\alpha_{24}}{\alpha_{23}}+2\frac{\alpha_{24}\cdot\alpha_{34}}{\alpha_{23}}+\\
&\qquad+2\frac{\alpha_{12}\cdot\alpha_{23}}{\rho}+2\frac{\alpha_{12}\cdot\alpha_{23}}{\alpha_{34}}+2\frac{\rho\cdot\alpha_{34}}{\alpha_{12}}+2\frac{\alpha_{12}\cdot\alpha_{24}}{\alpha_{34}}+\\
&\qquad\left.+2\frac{\alpha_{24}\cdot\alpha_{23}}{\alpha_{34}}+2\frac{\alpha_{34}\cdot\alpha_{23}}{\phi}\right\}+\mathcal{O}(\alpha'^2)\,, \qquad \text{(A.23)}
\end{aligned}
$$

$$
\begin{aligned}
L_3 =\;& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\rho\cdot\alpha_{12}}+\frac{1}{\alpha_{34}\cdot\alpha_{12}}\right\}-\frac{\pi^2}{6}\left\{\frac{\alpha_{23}}{\alpha_{34}}+\frac{\alpha_{23}}{\rho}+\frac{\rho}{\alpha_{12}}+\frac{\alpha_{34}}{\alpha_{12}}+\frac{\alpha_{24}}{\alpha_{34}}\right\}+\\
&+2\zeta(3)\alpha'\left\{\alpha_{24}+\frac{\alpha_{24}^2}{\alpha_{34}}+2\frac{\alpha_{24}\cdot\alpha_{23}}{\alpha_{34}}+2\frac{\rho\cdot\alpha_{34}}{\alpha_{12}}+\frac{\alpha_{12}\cdot\alpha_{23}}{\rho}+\frac{\alpha_{34}^2}{\alpha_{12}}+\frac{\alpha_{23}^2}{\rho}+\right.\\
&\qquad\left.\frac{\alpha_{12}\cdot\alpha_{23}}{\alpha_{34}}+\frac{\alpha_{12}\cdot\alpha_{24}}{\alpha_{34}}+\frac{\alpha_{23}^2}{\alpha_{34}}+\frac{\rho^2}{\alpha_{12}}\right\}+\mathcal{O}(\alpha'^2)\,, \qquad \text{(A.24)}
\end{aligned}
$$

$$
\begin{aligned}
{L_3}' =\;& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\phi\cdot\alpha_{34}}+\frac{1}{\alpha_{12}\cdot\alpha_{34}}\right\}-\frac{\pi^2}{6}\left\{\frac{\alpha_{23}}{\alpha_{12}}+\frac{\alpha_{23}}{\phi}+\frac{\phi}{\alpha_{34}}+\frac{\alpha_{12}}{\alpha_{34}}+\frac{\alpha_{13}}{\alpha_{12}}\right\}+\\
&+2\zeta(3)\alpha'\left\{\alpha_{13}+\frac{\alpha_{13}^2}{\alpha_{12}}+2\frac{\alpha_{13}\cdot\alpha_{23}}{\alpha_{12}}+2\frac{\phi\cdot\alpha_{12}}{\alpha_{34}}+\frac{\alpha_{34}\cdot\alpha_{23}}{\phi}+\frac{\alpha_{12}^2}{\alpha_{34}}+\frac{\alpha_{23}^2}{\phi}+\right.\\
&\qquad\left.+\frac{\alpha_{34}\cdot\alpha_{23}}{\alpha_{12}}+\frac{\alpha_{34}\cdot\alpha_{13}}{\alpha_{12}}+\frac{\alpha_{23}^2}{\alpha_{12}}+\frac{\phi^2}{\alpha_{34}}\right\}+\mathcal{O}(\alpha'^2)\,, \qquad \text{(A.25)}
\end{aligned}
$$

$$
\begin{aligned}
L_4 =\;& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\alpha_{23}\cdot\phi}+\frac{1}{\alpha_{23}\cdot\alpha_{12}}-\frac{\alpha_{13}}{\alpha_{23}\cdot\rho\cdot\alpha_{12}}\right\}+ \qquad \text{(A.26)}\\
&+\frac{\pi^2}{6}\left\{\frac{\alpha_{13}}{\rho}-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{13}}{\alpha_{23}}-\frac{\alpha_{12}}{\alpha_{23}}-\frac{\phi}{\alpha_{23}}\right\}+\\
&+2\zeta(3)\alpha'\left\{2\alpha_{13}+\frac{\alpha_{34}\cdot\alpha_{23}}{\phi}-\frac{\alpha_{13}\cdot\alpha_{12}}{\rho}+\frac{\alpha_{34}\cdot\alpha_{23}}{\alpha_{12}}+\frac{\alpha_{13}\cdot\rho}{\alpha_{23}}+\frac{\alpha_{13}\cdot\alpha_{12}}{\alpha_{23}}+\right.\\
&\qquad-\frac{\alpha_{23}\cdot\alpha_{13}}{\rho}+2\frac{\alpha_{34}\cdot\alpha_{13}}{\alpha_{12}}+3\frac{\alpha_{34}\cdot\alpha_{13}}{\alpha_{23}}+2\frac{\alpha_{13}\cdot\alpha_{24}}{\alpha_{23}}+2\frac{\phi\cdot\alpha_{12}}{\alpha_{23}}+\\
&\qquad\left.+\frac{\phi^2}{\alpha_{23}}+\frac{\alpha_{34}^2}{\phi}+\frac{\alpha_{34}^2}{\alpha_{12}}-\frac{\alpha_{13}\cdot\rho\cdot\alpha_{34}}{\alpha_{23}\cdot\alpha_{12}}+\frac{\alpha_{34}\cdot\alpha_{13}^2}{\alpha_{23}\cdot\alpha_{12}}+\frac{\alpha_{12}^2}{\alpha_{23}}\right\}+\mathcal{O}(\alpha'^2)\,,
\end{aligned}
$$

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$$
\begin{aligned}
L_4{}' =& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\alpha_{23}\cdot\rho}+\frac{1}{\alpha_{23}\cdot\alpha_{34}}-\frac{\alpha_{24}}{\alpha_{23}\cdot\phi\cdot\alpha_{34}}\right\}+\\
&+\frac{\pi^2}{6}\left\{\frac{\alpha_{24}}{\phi}-\frac{\alpha_{12}}{\rho}-\frac{\alpha_{12}}{\alpha_{34}}-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\rho}{\alpha_{23}}\right\}+\\
&+2\zeta(3)\alpha'\Bigg\{2\alpha_{24}+\frac{\alpha_{12}\cdot\alpha_{23}}{\rho}-\frac{\alpha_{24}\cdot\alpha_{34}}{\phi}+\frac{\alpha_{12}\cdot\alpha_{23}}{\alpha_{34}}+\frac{\alpha_{24}\cdot\phi}{\alpha_{23}}+\frac{\alpha_{24}\cdot\alpha_{34}}{\alpha_{23}}-\\
&\qquad-\frac{\alpha_{23}\cdot\alpha_{24}}{\phi}+2\frac{\alpha_{12}\cdot\alpha_{24}}{\alpha_{34}}+3\frac{\alpha_{12}\cdot\alpha_{24}}{\alpha_{23}}+2\frac{\alpha_{24}\cdot\alpha_{13}}{\alpha_{23}}+2\frac{\rho\cdot\alpha_{34}}{\alpha_{23}}+\frac{\rho^2}{\alpha_{23}}+\\
&\qquad+\frac{\alpha_{12}^2}{\rho}+\frac{\alpha_{12}^2}{\alpha_{34}}-\frac{\alpha_{24}\cdot\phi\cdot\alpha_{12}}{\alpha_{23}\cdot\alpha_{34}}+\frac{\alpha_{12}\cdot\alpha_{24}^2}{\alpha_{23}\cdot\alpha_{34}}+\frac{\alpha_{34}^2}{\alpha_{23}}\Bigg\}+\mathcal{O}(\alpha'^2)\,, \qquad (A.27)\\
L_7 =& \frac{1}{(2\alpha')^2}\left\{\frac{1}{\rho\cdot\alpha_{23}}+\frac{1}{\phi\cdot\alpha_{23}}\right\}-\frac{\pi^2}{6}\left\{\frac{\alpha_{34}}{\alpha_{23}}+\frac{\rho}{\alpha_{23}}+\frac{\alpha_{34}}{\phi}+\frac{\alpha_{12}}{\rho}+\frac{\alpha_{24}}{\alpha_{23}}\right\}+\\
&+2\zeta(3)\alpha'\Bigg\{2\alpha_{34}-2\alpha_{12}+2\alpha_{24}+2\rho+\frac{\alpha_{34}\cdot\alpha_{23}}{\phi}+2\frac{\alpha_{34}^2}{\alpha_{23}}+\frac{\rho^2}{\alpha_{23}}+\frac{\alpha_{34}^2}{\phi}+\frac{\phi\cdot\alpha_{12}}{\alpha_{23}}-\\
&\qquad-\frac{\alpha_{34}\cdot\phi}{\alpha_{23}}+2\frac{\rho\cdot\alpha_{24}}{\alpha_{23}}+2\frac{\rho\cdot\alpha_{34}}{\alpha_{23}}-\frac{\alpha_{12}\cdot\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{34}\cdot\alpha_{12}}{\alpha_{23}}+3\frac{\alpha_{34}\cdot\alpha_{24}}{\alpha_{23}}+\\
&\qquad+\frac{\alpha_{23}\cdot\alpha_{12}}{\rho}+\frac{\alpha_{12}^2}{\rho}+\frac{\alpha_{24}^2}{\alpha_{23}}\Bigg\}+\mathcal{O}(\alpha'^2)\,. \qquad (A.28)
\end{aligned}
$$

Besides the α_{ij} and ρ variables, respectively defined in eqs. (A.2) and (A.9), the ϕ variable appearing in many of the kinematic factors is defined as

$$
\phi = \alpha_{23}+\alpha_{24}+\alpha_{34}\,. \qquad (A.29)
$$

On this list, the expressions for K_6, L_5 and L_6 have been omitted since, using the relations in (A.6), they can be directly obtained in terms of kinematic factors that do appear on the list. In fact, using these relations, K_6, L_5 and L_6 have been already eliminated in the expression for $A(1,2,3,4,5)$ that comes in (5.29) . A remarkable fact, present in every kinematic factor, is that there is no $1/\alpha'$ contribution to it.

B. Contributions to the five gluon tree amplitude, at higher orders in α'

Using the explicit expressions of the kinematic factors (up to $\mathcal{O}(\alpha')$ terms) given in the previous subsection, and substituting them in eq. (5.29), leads directly to the expressions of $A^{(0)}(1,2,3,4,5)$, $A^{(2)}(1,2,3,4,5)$ and $A^{(3)}(1,2,3,4,5)$ of eq. (5.2). Since the expression for $A^{(0)}(1,2,3,4,5)$ was already presented in eq. (5.3), here we just give the following expressions for $A^{(2)}(1,2,3,4,5)$ and $A^{(3)}(1,2,3,4,5)$:

$$
\begin{aligned}
A^{(2)}(1,2,3,4,5) =& \frac{4\pi^2 g^3}{3}\times\\
&\times\Bigg[(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)\Bigg\{(\zeta_5\cdot k_1)\alpha_{23}\Bigg\{-\frac{\alpha_{24}}{\alpha_{34}}-\frac{\alpha_{23}}{\alpha_{34}}-\frac{\rho}{\alpha_{12}}-\frac{\alpha_{12}}{\alpha_{34}}-\\
&\qquad-\frac{\alpha_{23}}{\phi}+1-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{23}}{\rho}-\\
&\qquad-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\rho}{\alpha_{23}}-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}\Bigg\}-
\end{aligned}
$$

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$$
\begin{aligned}
&-(\zeta_5\cdot k_2)\alpha_{13}\Big\{-\frac{\alpha_{23}}{\alpha_{34}}-\frac{\alpha_{23}}{\rho}-\frac{\rho}{\alpha_{12}}-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{24}}{\alpha_{34}}\Big\}+\\
&+(\zeta_5\cdot k_3)\alpha_{24}\Big\{-\frac{\phi}{\alpha_{34}}-\frac{\alpha_{12}}{\alpha_{34}}-\frac{\alpha_{23}}{\alpha_{12}}-\frac{\alpha_{23}}{\phi}-\frac{\alpha_{13}}{\alpha_{12}}\Big\}+\\
&+(\zeta_5\cdot k_3)\alpha_{23}\Big\{-\frac{\alpha_{24}}{\alpha_{34}}-\frac{\alpha_{23}}{\alpha_{34}}-\frac{\rho}{\alpha_{12}}-\frac{\alpha_{12}}{\alpha_{34}}-\\
&\qquad\qquad-\frac{\alpha_{23}}{\phi}+1-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{23}}{\rho}-\\
&\qquad\qquad-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\rho}{\alpha_{23}}-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}\Big\}\Big\}+\\
&+(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_4)\Big\{-(\zeta_5\cdot k_1)\alpha_{23}\Big\{-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\rho}{\alpha_{23}}-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}\Big\}+\\
&\qquad+(\zeta_5\cdot k_2)\alpha_{34}\Big\{1-\frac{\alpha_{12}}{\alpha_{34}}-\frac{\alpha_{23}}{\phi}\Big\}-\\
&\qquad-(\zeta_5\cdot k_2)\alpha_{23}\Big\{-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\rho}{\alpha_{23}}-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}\Big\}-\\
&\qquad-(\zeta_5\cdot k_3)\alpha_{12}\Big\{1-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{23}}{\rho}\Big\}\Big\}+\\
&+(\zeta_1\cdot\zeta_4)(\zeta_2\cdot\zeta_3)\Big\{(\zeta_5\cdot k_1)\alpha_{34}\Big\{-\frac{\alpha_{13}}{\alpha_{23}}-\frac{\alpha_{12}}{\alpha_{23}}-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{23}}{\phi}-\frac{\alpha_{12}}{\alpha_{34}}\Big\}-\\
&\qquad-(\zeta_5\cdot k_1)\alpha_{13}\Big\{-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}-\frac{\alpha_{24}}{\alpha_{23}}\Big\}-\\
&\qquad-(\zeta_5\cdot k_2)\alpha_{13}\Big\{-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}-\frac{\alpha_{24}}{\alpha_{23}}\Big\}+\\
&\qquad+(\zeta_5\cdot k_3)\alpha_{12}\Big\{-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}-\frac{\alpha_{23}}{\rho}-\frac{\alpha_{34}}{\alpha_{12}}\Big\}\Big\}\Big]+\\
&+\Big[(\zeta_2\cdot\zeta_3)\Big\{(\zeta_5\cdot k_2)\Big(\!(\zeta_1\cdot k_3)(\zeta_4\cdot k_1)\Big\{-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}-\frac{\alpha_{24}}{\alpha_{23}}\Big\}+\\
&\qquad+(\zeta_1\cdot k_3)(\zeta_4\cdot k_3)\Big\{-\frac{\alpha_{12}}{\alpha_{34}}+\frac{\alpha_{24}}{\phi}-\frac{\rho}{\alpha_{23}}-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}\Big\}+\\
&\qquad+(\zeta_1\cdot k_2)(\zeta_4\cdot k_3)\Big\{-\frac{\alpha_{24}}{\alpha_{34}}-\frac{\alpha_{23}}{\alpha_{34}}-\frac{\rho}{\alpha_{12}}-\frac{\alpha_{12}}{\alpha_{34}}-\\
&\qquad\qquad-\frac{\alpha_{23}}{\phi}+1-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{23}}{\rho}-\\
&\qquad\qquad-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\rho}{\alpha_{23}}-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}\Big\}+\\
&\qquad+(\zeta_1\cdot k_4)(\zeta_4\cdot k_3)\Big\{-\frac{\alpha_{13}}{\alpha_{23}}-\frac{\alpha_{12}}{\alpha_{23}}-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{23}}{\phi}-\frac{\alpha_{12}}{\alpha_{34}}\Big\}\Big)-\\
&\quad-(\zeta_5\cdot k_3)\Big(\!(\zeta_1\cdot k_2)(\zeta_4\cdot k_1)\Big\{-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}-\frac{\alpha_{23}}{\rho}-\frac{\alpha_{34}}{\alpha_{12}}\Big\}\\
&\qquad+(\zeta_1\cdot k_3)(\zeta_4\cdot k_2)\Big\{-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\rho}{\alpha_{23}}-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}\Big\}+\\
&\qquad+(\zeta_1\cdot k_2)(\zeta_4\cdot k_2)\Big\{-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{34}}{\phi}+\frac{\alpha_{13}}{\rho}-\frac{\phi}{\alpha_{23}}-\frac{\alpha_{13}}{\alpha_{23}}-\frac{\alpha_{12}}{\alpha_{23}}\Big\}+\\
&\qquad+(\zeta_1\cdot k_4)(\zeta_4\cdot k_2)\Big\{-\frac{\alpha_{12}}{\alpha_{23}}-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{13}}{\alpha_{23}}\Big\}\Big)+
\end{aligned}
$$

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$$
\begin{aligned}
&+(\zeta_5\cdot k_1)\Bigg((\zeta_1\cdot k_2)(\zeta_4\cdot k_3)\Bigg\{-\frac{\alpha_{24}}{\alpha_{34}}-\frac{\alpha_{23}}{\alpha_{34}}-\frac{\rho}{\alpha_{12}}-\frac{\alpha_{12}}{\alpha_{34}}-\\
&\qquad-\frac{\alpha_{23}}{\phi}+1-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{23}}{\rho}-\\
&\qquad-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\rho}{\alpha_{23}}-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}\Bigg\}-\\
&\qquad-(\zeta_1\cdot k_3)(\zeta_4\cdot k_2)\left\{-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\rho}{\alpha_{23}}-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}\right\}\Bigg)\Bigg\}+\\
&+(\zeta_1\cdot\zeta_4)\Bigg\{-(\zeta_5\cdot k_1)\Bigg((\zeta_2\cdot k_3)(\zeta_3\cdot k_4)\left\{-\frac{\alpha_{13}}{\alpha_{23}}-\frac{\alpha_{12}}{\alpha_{23}}-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{23}}{\phi}-\frac{\alpha_{12}}{\alpha_{34}}\right\}-\\
&\qquad-(\zeta_2\cdot k_4)(\zeta_3\cdot k_2)\left\{-\frac{\alpha_{12}}{\alpha_{23}}-\frac{\alpha_{34}}{\phi}-\frac{\alpha_{13}}{\alpha_{23}}\right\}+\\
&\qquad+(\zeta_2\cdot k_4)(\zeta_3\cdot k_4)\left\{-\frac{\alpha_{12}}{\alpha_{34}}-\frac{\alpha_{23}}{\phi}\right\}-\\
&\qquad-(\zeta_2\cdot k_1)(\zeta_3\cdot k_4)\left\{-\frac{\rho}{\alpha_{12}}-\frac{\alpha_{23}}{\alpha_{34}}-\frac{\alpha_{24}}{\alpha_{34}}\right\}\Bigg)-\\
&\quad-(\zeta_5\cdot k_4)\Bigg((\zeta_2\cdot k_1)(\zeta_3\cdot k_2)\left\{-\frac{\alpha_{24}}{\alpha_{23}}-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}-\frac{\alpha_{23}}{\rho}-\frac{\alpha_{34}}{\alpha_{12}}\right\}+\\
&\qquad+(\zeta_2\cdot k_1)(\zeta_3\cdot k_1)\left\{-\frac{\alpha_{34}}{\alpha_{12}}-\frac{\alpha_{23}}{\rho}\right\}-\\
&\qquad-(\zeta_2\cdot k_3)(\zeta_3\cdot k_1)\left\{-\frac{\alpha_{34}}{\alpha_{23}}-\frac{\alpha_{12}}{\rho}-\frac{\alpha_{24}}{\alpha_{23}}\right\}-\\
&\qquad-(\zeta_2\cdot k_4)(\zeta_3\cdot k_1)\Bigg)+\\
&\quad+(\zeta_5\cdot k_2)\Bigg((\zeta_2\cdot k_1)(\zeta_3\cdot k_4)\left\{-\frac{\rho}{\alpha_{12}}-\frac{\alpha_{23}}{\alpha_{34}}-\frac{\alpha_{24}}{\alpha_{34}}\right\}-\\
&\qquad-(\zeta_2\cdot k_4)(\zeta_3\cdot k_1)\Bigg)\Bigg\}\Bigg]+\\
&+(\text{cyclic permutations of indexes }(1,2,3,4,5))
\end{aligned}
\tag{B.1}
$$

and

$$
\begin{aligned}
&A^{(3)}(1,2,3,4,5)=16\zeta(3)g^3\times\\
&\times\Bigg[(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4)\Bigg\{(\zeta_5\cdot k_1)\alpha_{23}\Bigg\{\frac{{\alpha_{34}}^2}{\alpha_{12}}+3\ \alpha_{24}+\frac{{\alpha_{12}}^2}{\alpha_{34}}+\frac{2\,\rho\ \alpha_{24}}{\alpha_{23}}+\\
&\qquad+\frac{2\ \alpha_{12}\,\alpha_{24}}{\alpha_{34}}+\frac{2\,\rho\ \alpha_{34}}{\alpha_{23}}+\frac{2\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\frac{\rho^2}{\alpha_{12}}+\\
&\qquad+\frac{{\alpha_{34}}^2}{\phi}+\frac{{\alpha_{23}}^2}{\rho}+\frac{2\ \alpha_{23}\,\alpha_{24}}{\alpha_{34}}+\frac{2\ \alpha_{12}\,\alpha_{23}}{\alpha_{34}}+\\
&\qquad+\frac{2\ \alpha_{34}\,\alpha_{23}}{\phi}+\frac{2\ \alpha_{12}\,\alpha_{23}}{\rho}+\frac{2\,\rho\ \alpha_{34}}{\alpha_{12}}-\phi+\\
&\qquad+\frac{\rho^2}{\alpha_{23}}+\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{23}}^2}{\alpha_{34}}+\frac{{\alpha_{24}}^2}{\alpha_{34}}+\\
&\qquad+\frac{{\alpha_{34}}^2}{\alpha_{23}}+\frac{{\alpha_{24}}^2}{\alpha_{23}}+\frac{{\alpha_{23}}^2}{\phi}\Bigg\}-\\
&\quad-(\zeta_5\cdot k_2)\alpha_{13}\Bigg\{\frac{\alpha_{12}\,\alpha_{23}}{\alpha_{34}}+\frac{\alpha_{12}\,\alpha_{23}}{\rho}+\frac{2\ \alpha_{34}\rho}{\alpha_{12}}+\frac{{\alpha_{23}}^2}{\alpha_{34}}+
\end{aligned}
$$

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$$
\begin{aligned}
&+\frac{{\alpha_{23}}^2}{\rho}+\frac{{\alpha_{34}}^2}{\alpha_{12}}+\frac{\rho^2}{\alpha_{12}}+\frac{2\;\alpha_{23}\,\alpha_{24}}{\alpha_{34}}+\\
&+\frac{{\alpha_{24}}^2}{\alpha_{34}}+\frac{\alpha_{12}\,\alpha_{24}}{\alpha_{34}}+\;\alpha_{24}\Bigg\}+\\
+(\zeta_5\cdot k_3)\alpha_{24}\Bigg\{&\frac{\phi^2}{\alpha_{34}}+\frac{\alpha_{34}\,\alpha_{23}}{\alpha_{12}}+\frac{\alpha_{34}\,\alpha_{23}}{\phi}+\frac{2\,\phi\;\alpha_{12}}{\alpha_{34}}+\\
&+\frac{{\alpha_{23}}^2}{\alpha_{12}}+\frac{{\alpha_{23}}^2}{\phi}+\frac{{\alpha_{12}}^2}{\alpha_{34}}+\frac{2\;\alpha_{23}\,\alpha_{13}}{\alpha_{12}}+\\
&+\frac{{\alpha_{13}}^2}{\alpha_{12}}+\frac{\alpha_{34}\,\alpha_{13}}{\alpha_{12}}+\;\alpha_{13}\Bigg\}+\\
+(\zeta_5\cdot k_3)\alpha_{23}\Bigg\{&\frac{{\alpha_{34}}^2}{\alpha_{12}}+3\;\alpha_{24}+\frac{{\alpha_{12}}^2}{\alpha_{34}}+\frac{2\,\rho\;\alpha_{24}}{\alpha_{23}}+\\
&+\frac{2\;\alpha_{12}\,\alpha_{24}}{\alpha_{34}}+\frac{2\,\rho\;\alpha_{34}}{\alpha_{23}}+\frac{2\;\alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\frac{\rho^2}{\alpha_{12}}+\\
&+\frac{{\alpha_{34}}^2}{\phi}+\frac{{\alpha_{23}}^2}{\rho}+\frac{2\;\alpha_{23}\,\alpha_{24}}{\alpha_{34}}+\frac{2\;\alpha_{12}\,\alpha_{23}}{\alpha_{34}}+\\
&+\frac{2\;\alpha_{34}\,\alpha_{23}}{\phi}+\frac{2\;\alpha_{12}\,\alpha_{23}}{\rho}+\frac{2\,\rho\;\alpha_{34}}{\alpha_{12}}-\phi+\\
&+\frac{\rho^2}{\alpha_{23}}+\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{23}}^2}{\alpha_{34}}+\frac{{\alpha_{24}}^2}{\alpha_{34}}+\\
&+\frac{{\alpha_{34}}^2}{\alpha_{23}}+\frac{{\alpha_{24}}^2}{\alpha_{23}}+\frac{{\alpha_{23}}^2}{\phi}\Bigg\}\Bigg\}+\\
+(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_4)\Bigg\{-(\zeta_5\cdot k_1)\alpha_{23}\Bigg\{&\frac{\alpha_{12}\,\alpha_{23}}{\rho}-\frac{\alpha_{34}\phi}{\alpha_{23}}-\frac{\alpha_{34}\,\alpha_{12}}{\alpha_{23}}-\frac{\alpha_{12}\,\alpha_{24}}{\alpha_{23}}+\\
&+\frac{\phi\;\alpha_{12}}{\alpha_{23}}+\frac{2\,\rho\;\alpha_{24}}{\alpha_{23}}+\frac{2\,\rho\;\alpha_{34}}{\alpha_{23}}+\frac{3\;\alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\\
&+2\;\alpha_{24}-2\;\alpha_{12}+\frac{{\alpha_{24}}^2}{\alpha_{23}}+\frac{2\;{\alpha_{34}}^2}{\alpha_{23}}+\\
&+\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{34}}^2}{\phi}+\frac{\rho^2}{\alpha_{23}}+\frac{\alpha_{34}\,\alpha_{23}}{\phi}+2\;\alpha_{34}+2\,\rho\Bigg\}+\\
+(\zeta_5\cdot k_2)\alpha_{34}\Bigg\{&\frac{{\alpha_{12}}^2}{\alpha_{34}}+\frac{{\alpha_{23}}^2}{\phi}+\frac{\alpha_{34}\,\alpha_{23}}{\phi}+\frac{\phi\;\alpha_{12}}{\alpha_{34}}-\\
&-\phi-2\;\alpha_{12}-\;\alpha_{34}-2\;\alpha_{23}-2\;\alpha_{13}\Bigg\}-\\
-(\zeta_5\cdot k_2)\alpha_{23}\Bigg\{&\frac{\alpha_{12}\,\alpha_{23}}{\rho}-\frac{\alpha_{34}\phi}{\alpha_{23}}-\frac{\alpha_{34}\,\alpha_{12}}{\alpha_{23}}-\frac{\alpha_{12}\,\alpha_{24}}{\alpha_{23}}+\\
&+\frac{\phi\;\alpha_{12}}{\alpha_{23}}+\frac{2\,\rho\;\alpha_{24}}{\alpha_{23}}+\frac{2\,\rho\;\alpha_{34}}{\alpha_{23}}+\frac{3\;\alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\\
&+2\;\alpha_{24}-2\;\alpha_{12}+\frac{{\alpha_{24}}^2}{\alpha_{23}}+\frac{2\;{\alpha_{34}}^2}{\alpha_{23}}+\\
&+\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{34}}^2}{\phi}+\frac{\rho^2}{\alpha_{23}}+\frac{\alpha_{34}\,\alpha_{23}}{\phi}+2\;\alpha_{34}+2\,\rho\Bigg\}-\\
-(\zeta_5\cdot k_3)\alpha_{12}\Bigg\{&\frac{{\alpha_{34}}^2}{\alpha_{12}}+\frac{{\alpha_{23}}^2}{\rho}+\frac{\alpha_{12}\,\alpha_{23}}{\rho}+\frac{\rho\;\alpha_{34}}{\alpha_{12}}-\\
&-\rho-2\;\alpha_{34}-\;\alpha_{12}-2\;\alpha_{23}-2\;\alpha_{24}\Bigg\}\Bigg\}+
\end{aligned}
$$

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$$
\begin{aligned}
&+(\zeta_1\cdot\zeta_4)(\zeta_2\cdot\zeta_3)\Bigg\{(\zeta_5\cdot k_1)\alpha_{34}\Bigg\{\frac{{\alpha_{13}}^2}{\alpha_{23}}+\frac{2\ \alpha_{12}\,\alpha_{13}}{\alpha_{23}}+\frac{\phi\ \alpha_{12}}{\alpha_{34}}+\frac{2\ \alpha_{34}\,\alpha_{23}}{\phi}+\\
&\qquad+\frac{{\alpha_{12}}^2}{\alpha_{23}}+\frac{{\alpha_{34}}^2}{\phi}+\frac{\phi\ \alpha_{13}}{\alpha_{23}}+\\
&\qquad+\frac{\phi\ \alpha_{12}}{\alpha_{23}}+\frac{{\alpha_{23}}^2}{\phi}+\alpha_{13}+\frac{{\alpha_{12}}^2}{\alpha_{34}}\Bigg\}-\\
&\quad-(\zeta_5\cdot k_1)\alpha_{13}\Bigg\{\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{34}}^2}{\alpha_{23}}+\frac{\rho\ \alpha_{34}}{\alpha_{23}}+\frac{\rho\ \alpha_{24}}{\alpha_{23}}+\\
&\qquad+\frac{2\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\frac{\alpha_{12}\,\alpha_{23}}{\rho}+\frac{{\alpha_{24}}^2}{\alpha_{23}}+2\ \alpha_{24}\Bigg\}-\\
&\quad-(\zeta_5\cdot k_2)\alpha_{13}\Bigg\{\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{34}}^2}{\alpha_{23}}+\frac{\rho\ \alpha_{34}}{\alpha_{23}}+\frac{\rho\ \alpha_{24}}{\alpha_{23}}+\\
&\qquad+\frac{2\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\frac{\alpha_{12}\,\alpha_{23}}{\rho}+\frac{{\alpha_{24}}^2}{\alpha_{23}}+2\ \alpha_{24}\Bigg\}+\\
&\quad+(\zeta_5\cdot k_3)\alpha_{12}\Bigg\{\frac{{\alpha_{24}}^2}{\alpha_{23}}+\frac{2\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\frac{\rho\ \alpha_{34}}{\alpha_{12}}+\frac{2\ \alpha_{12}\,\alpha_{23}}{\rho}+\\
&\qquad+\frac{{\alpha_{34}}^2}{\alpha_{23}}+\frac{{\alpha_{12}}^2}{\rho}+\frac{\rho\ \alpha_{24}}{\alpha_{23}}+\frac{\rho\ \alpha_{34}}{\alpha_{23}}+\\
&\qquad+\frac{{\alpha_{23}}^2}{\rho}+\alpha_{24}+\frac{{\alpha_{34}}^2}{\alpha_{12}}\Bigg\}\Bigg\}\Bigg]+\\
&+\Bigg[(\zeta_2\cdot\zeta_3)\Bigg\{(\zeta_5\cdot k_2)\Bigg((\zeta_1\cdot k_3)(\zeta_4\cdot k_1)\Bigg\{\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{34}}^2}{\alpha_{23}}+\frac{\rho\ \alpha_{34}}{\alpha_{23}}+\frac{\rho\ \alpha_{24}}{\alpha_{23}}+\\
&\qquad+\frac{2\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\frac{\alpha_{12}\,\alpha_{23}}{\rho}+\frac{{\alpha_{24}}^2}{\alpha_{23}}+2\ \alpha_{24}\Bigg\}+\\
&\quad+(\zeta_1\cdot k_3)(\zeta_4\cdot k_3)\Bigg\{\frac{{\alpha_{12}}^2}{\alpha_{34}}+\frac{\alpha_{12}\,\alpha_{23}}{\rho}+\frac{\alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\frac{2\,\rho\ \alpha_{34}}{\alpha_{23}}+\\
&\qquad+2\ \alpha_{24}+\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{34}}^2}{\alpha_{23}}+\frac{\rho^2}{\alpha_{23}}+\\
&\qquad+\frac{2\ \alpha_{13}\,\alpha_{24}}{\alpha_{23}}+\frac{\alpha_{24}\phi}{\alpha_{23}}+\frac{\alpha_{12}\,\alpha_{23}}{\alpha_{34}}+\frac{3\ \alpha_{12}\,\alpha_{24}}{\alpha_{23}}-\\
&\qquad-\frac{\alpha_{23}\,\alpha_{24}}{\phi}-\frac{\alpha_{34}\,\alpha_{24}}{\phi}-\frac{\alpha_{24}\phi\ \alpha_{12}}{\alpha_{23}\,\alpha_{34}}+\\
&\qquad+\frac{\alpha_{12}\,{\alpha_{24}}^2}{\alpha_{23}\,\alpha_{34}}+\frac{2\ \alpha_{12}\,\alpha_{24}}{\alpha_{34}}\Bigg\}+\\
&\quad+(\zeta_1\cdot k_2)(\zeta_4\cdot k_3)\Bigg\{\frac{{\alpha_{34}}^2}{\alpha_{12}}+3\ \alpha_{24}+\frac{{\alpha_{12}}^2}{\alpha_{34}}+\frac{2\,\rho\ \alpha_{24}}{\alpha_{23}}+\\
&\qquad+\frac{2\ \alpha_{12}\,\alpha_{24}}{\alpha_{34}}+\frac{2\,\rho\ \alpha_{34}}{\alpha_{23}}+\frac{2\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\frac{\rho^2}{\alpha_{12}}+\\
&\qquad+\frac{{\alpha_{34}}^2}{\phi}+\frac{{\alpha_{23}}^2}{\rho}+\frac{2\ \alpha_{23}\,\alpha_{24}}{\alpha_{34}}+\frac{2\ \alpha_{12}\,\alpha_{23}}{\alpha_{34}}+\\
&\qquad+\frac{2\ \alpha_{34}\,\alpha_{23}}{\phi}+\frac{2\ \alpha_{12}\,\alpha_{23}}{\rho}+\frac{2\,\rho\ \alpha_{34}}{\alpha_{12}}-\phi+\\
&\qquad+\frac{\rho^2}{\alpha_{23}}+\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{23}}^2}{\alpha_{34}}+\frac{{\alpha_{24}}^2}{\alpha_{34}}+\\
&\qquad+\frac{{\alpha_{34}}^2}{\alpha_{23}}+\frac{{\alpha_{24}}^2}{\alpha_{23}}+\frac{{\alpha_{23}}^2}{\phi}\Bigg\}
\end{aligned}
$$

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$$
\begin{aligned}
&+ (\zeta_1\cdot k_4)(\zeta_4\cdot k_3)\bigg\{\frac{{\alpha_{13}}^2}{\alpha_{23}} + \frac{2\ \alpha_{12}\,\alpha_{13}}{\alpha_{23}} + \frac{\phi\ \alpha_{12}}{\alpha_{34}} + \frac{2\ \alpha_{34}\,\alpha_{23}}{\phi} + \\
&\qquad + \frac{{\alpha_{12}}^2}{\alpha_{23}} + \frac{{\alpha_{34}}^2}{\phi} + \frac{\phi\ \alpha_{13}}{\alpha_{23}} + \\
&\qquad + \frac{\phi\ \alpha_{12}}{\alpha_{23}} + \frac{{\alpha_{23}}^2}{\phi} + \alpha_{13} + \frac{{\alpha_{12}}^2}{\alpha_{34}}\bigg\}\bigg) \\
&- (\zeta_5\cdot k_3)\bigg((\zeta_1\cdot k_2)(\zeta_4\cdot k_1)\bigg\{\frac{{\alpha_{24}}^2}{\alpha_{23}} + \frac{2\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}} + \frac{\rho\ \alpha_{34}}{\alpha_{12}} + \frac{2\ \alpha_{12}\,\alpha_{23}}{\rho} + \\
&\qquad + \frac{{\alpha_{34}}^2}{\alpha_{23}} + \frac{{\alpha_{12}}^2}{\rho} + \frac{\rho\ \alpha_{24}}{\alpha_{23}} + \frac{\rho\ \alpha_{34}}{\alpha_{23}} + \\
&\qquad + \frac{{\alpha_{23}}^2}{\rho} + \alpha_{24} + \frac{{\alpha_{34}}^2}{\alpha_{12}}\bigg\} + \\
&\quad + (\zeta_1\cdot k_3)(\zeta_4\cdot k_2)\bigg\{\frac{\alpha_{12}\,\alpha_{23}}{\rho} - \frac{\alpha_{34}\phi}{\alpha_{23}} - \frac{\alpha_{34}\,\alpha_{12}}{\alpha_{23}} - \frac{\alpha_{12}\,\alpha_{24}}{\alpha_{23}} + \\
&\qquad + \frac{\phi\ \alpha_{12}}{\alpha_{23}} + \frac{2\,\rho\ \alpha_{24}}{\alpha_{23}} + \frac{2\,\rho\ \alpha_{34}}{\alpha_{23}} + \frac{3\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}} + \\
&\qquad + 2\ \alpha_{24} - 2\ \alpha_{12} + \frac{{\alpha_{24}}^2}{\alpha_{23}} + \frac{2\ {\alpha_{34}}^2}{\alpha_{23}} + \\
&\qquad + \frac{{\alpha_{12}}^2}{\rho} + \frac{{\alpha_{34}}^2}{\phi} + \frac{\rho^2}{\alpha_{23}} + \frac{\alpha_{34}\,\alpha_{23}}{\phi} + 2\ \alpha_{34} + 2\,\rho\bigg\} + \\
&\quad + (\zeta_1\cdot k_2)(\zeta_4\cdot k_2)\bigg\{2\alpha_{13} + \frac{\alpha_{34}\alpha_{23}}{\phi} - \frac{\alpha_{13}\alpha_{12}}{\rho} + \frac{\alpha_{34}\alpha_{23}}{\alpha_{12}} + \frac{\alpha_{13}\rho}{\alpha_{23}} + \\
&\qquad + \frac{\alpha_{13}\alpha_{12}}{\alpha_{23}} - \frac{\alpha_{23}\alpha_{13}}{\rho} + \frac{2\alpha_{34}\alpha_{13}}{\alpha_{12}} + \frac{3\alpha_{34}\alpha_{13}}{\alpha_{23}} + \\
&\qquad + \frac{2\alpha_{13}\alpha_{24}}{\alpha_{23}} + \frac{\phi\alpha_{12}}{\alpha_{23}} + \frac{\phi^2}{\alpha_{23}} + \frac{\alpha_{34}^2}{\phi} + \\
&\qquad + \frac{\alpha_{34}^2}{\alpha_{12}} - \frac{\alpha_{13}\rho\cdot\alpha_{34}}{\alpha_{23}\alpha_{12}} + \frac{\alpha_{34}\alpha_{13}^2}{\alpha_{23}\alpha_{12}} + \frac{\alpha_{12}^2}{\alpha_{23}}\bigg\} + \\
&\quad + (\zeta_1\cdot k_4)(\zeta_4\cdot k_2)\bigg\{\frac{{\alpha_{34}}^2}{\phi} + \frac{{\alpha_{12}}^2}{\alpha_{23}} + \frac{\phi\ \alpha_{12}}{\alpha_{23}} + \frac{\phi\ \alpha_{13}}{\alpha_{23}} + \\
&\qquad + \frac{2\ \alpha_{12}\,\alpha_{13}}{\alpha_{23}} + \frac{\alpha_{34}\,\alpha_{23}}{\phi} + \frac{{\alpha_{13}}^2}{\alpha_{23}} + 2\ \alpha_{13}\bigg\}\bigg) + \\
&+ (\zeta_5\cdot k_1)\bigg((\zeta_1\cdot k_2)(\zeta_4\cdot k_3)\bigg\{\frac{{\alpha_{34}}^2}{\alpha_{12}} + 3\ \alpha_{24} + \frac{{\alpha_{12}}^2}{\alpha_{34}} + \frac{2\,\rho\ \alpha_{24}}{\alpha_{23}} + \\
&\qquad + \frac{2\ \alpha_{12}\,\alpha_{24}}{\alpha_{34}} + \frac{2\,\rho\ \alpha_{34}}{\alpha_{23}} + \frac{2\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}} + \frac{\rho^2}{\alpha_{12}} + \\
&\qquad + \frac{{\alpha_{34}}^2}{\phi} + \frac{{\alpha_{23}}^2}{\rho} + \frac{2\ \alpha_{23}\,\alpha_{24}}{\alpha_{34}} + \frac{2\ \alpha_{12}\,\alpha_{23}}{\alpha_{34}} + \\
&\qquad + \frac{2\ \alpha_{34}\,\alpha_{23}}{\phi} + \frac{2\ \alpha_{12}\,\alpha_{23}}{\rho} + \frac{2\,\rho\ \alpha_{34}}{\alpha_{12}} - \phi + \\
&\qquad + \frac{\rho^2}{\alpha_{23}} + \frac{{\alpha_{12}}^2}{\rho} + \frac{{\alpha_{23}}^2}{\alpha_{34}} + \frac{{\alpha_{24}}^2}{\alpha_{34}} + \\
&\qquad + \frac{{\alpha_{34}}^2}{\alpha_{23}} + \frac{{\alpha_{24}}^2}{\alpha_{23}} + \frac{{\alpha_{23}}^2}{\phi}\bigg\} - \\
&\quad - (\zeta_1\cdot k_3)(\zeta_4\cdot k_2)\bigg\{\frac{\alpha_{12}\,\alpha_{23}}{\rho} - \frac{\alpha_{34}\phi}{\alpha_{23}} - \frac{\alpha_{34}\,\alpha_{12}}{\alpha_{23}} - \frac{\alpha_{12}\,\alpha_{24}}{\alpha_{23}} +
\end{aligned}
$$

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$$
\begin{aligned}
&+\frac{\phi\ \alpha_{12}}{\alpha_{23}}+\frac{2\,\rho\ \alpha_{24}}{\alpha_{23}}+\frac{2\,\rho\ \alpha_{34}}{\alpha_{23}}+\frac{3\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\\
&+2\ \alpha_{24}-2\ \alpha_{12}+\frac{{\alpha_{24}}^2}{\alpha_{23}}+\frac{2\ {\alpha_{34}}^2}{\alpha_{23}}+\\
&+\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{34}}^2}{\phi}+\frac{\rho^2}{\alpha_{23}}+\frac{\alpha_{34}\,\alpha_{23}}{\phi}+2\ \alpha_{34}+2\,\rho\Big\}\Big)\Big\}+\\
+(\zeta_1\cdot\zeta_4)\Big\{-(\zeta_5\cdot k_1)\Big(&(\zeta_2\cdot k_3)(\zeta_3\cdot k_4)\Big\{\frac{{\alpha_{13}}^2}{\alpha_{23}}+\frac{2\ \alpha_{12}\,\alpha_{13}}{\alpha_{23}}+\frac{\phi\ \alpha_{12}}{\alpha_{34}}+\frac{2\ \alpha_{34}\,\alpha_{23}}{\phi}+\\
&+\frac{{\alpha_{12}}^2}{\alpha_{23}}+\frac{{\alpha_{34}}^2}{\phi}+\frac{\phi\ \alpha_{13}}{\alpha_{23}}+\\
&+\frac{\phi\ \alpha_{12}}{\alpha_{23}}+\frac{{\alpha_{23}}^2}{\phi}+\alpha_{13}+\frac{{\alpha_{12}}^2}{\alpha_{34}}\Big\}-\\
&-(\zeta_2\cdot k_4)(\zeta_3\cdot k_2)\Big\{\frac{{\alpha_{34}}^2}{\phi}+\frac{{\alpha_{12}}^2}{\alpha_{23}}+\frac{\phi\ \alpha_{12}}{\alpha_{23}}+\frac{\phi\ \alpha_{13}}{\alpha_{23}}+\\
&+\frac{2\ \alpha_{12}\,\alpha_{13}}{\alpha_{23}}+\frac{\alpha_{34}\,\alpha_{23}}{\phi}+\frac{{\alpha_{13}}^2}{\alpha_{23}}+2\ \alpha_{13}\Big\}+\\
+(\zeta_2\cdot k_4)(\zeta_3\cdot k_4)\Big\{&-\alpha_{13}+\frac{\phi\ \alpha_{12}}{\alpha_{34}}+\frac{\alpha_{34}\,\alpha_{23}}{\phi}+\frac{{\alpha_{12}}^2}{\alpha_{34}}+\frac{{\alpha_{23}}^2}{\phi}\Big\}-\\
&-(\zeta_2\cdot k_1)(\zeta_3\cdot k_4)\Big\{2\ \alpha_{24}+\frac{\alpha_{34}\rho}{\alpha_{12}}+\frac{\rho^2}{\alpha_{12}}+\frac{{\alpha_{23}}^2}{\alpha_{34}}+\\
&+\frac{{\alpha_{24}}^2}{\alpha_{34}}+\frac{2\ \alpha_{23}\,\alpha_{24}}{\alpha_{34}}+\frac{\alpha_{23}\,\alpha_{12}}{\alpha_{34}}+\frac{\alpha_{12}\,\alpha_{24}}{\alpha_{34}}\Big\}\Big)-\\
-(\zeta_5\cdot k_4)\Big(&(\zeta_2\cdot k_1)(\zeta_3\cdot k_2)\Big\{\frac{{\alpha_{24}}^2}{\alpha_{23}}+\frac{2\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\frac{\rho\ \alpha_{34}}{\alpha_{12}}+\frac{2\ \alpha_{12}\,\alpha_{23}}{\rho}+\\
&+\frac{{\alpha_{34}}^2}{\alpha_{23}}+\frac{{\alpha_{12}}^2}{\rho}+\frac{\rho\ \alpha_{24}}{\alpha_{23}}+\frac{\rho\ \alpha_{34}}{\alpha_{23}}+\\
&+\frac{{\alpha_{23}}^2}{\rho}+\alpha_{24}+\frac{{\alpha_{34}}^2}{\alpha_{12}}\Big\}+\\
&+(\zeta_2\cdot k_1)(\zeta_3\cdot k_1)\Big\{-\alpha_{24}+\frac{\alpha_{34}\rho}{\alpha_{12}}+\frac{\alpha_{12}\,\alpha_{23}}{\rho}+\frac{{\alpha_{34}}^2}{\alpha_{12}}+\frac{{\alpha_{23}}^2}{\rho}\Big\}-\\
&-(\zeta_2\cdot k_3)(\zeta_3\cdot k_1)\Big\{\frac{{\alpha_{12}}^2}{\rho}+\frac{{\alpha_{34}}^2}{\alpha_{23}}+\frac{\rho\ \alpha_{34}}{\alpha_{23}}+\frac{\rho\ \alpha_{24}}{\alpha_{23}}+\\
&+\frac{2\ \alpha_{34}\,\alpha_{24}}{\alpha_{23}}+\frac{\alpha_{12}\,\alpha_{23}}{\rho}+\frac{{\alpha_{24}}^2}{\alpha_{23}}+2\ \alpha_{24}\Big\}-\\
&-(\zeta_2\cdot k_4)(\zeta_3\cdot k_1)\Big\{-\rho-2\ \alpha_{23}-2\ \alpha_{34}-\alpha_{12}-\alpha_{24}\Big\}\Big)+\\
+(\zeta_5\cdot k_2)\Big(&(\zeta_2\cdot k_1)(\zeta_3\cdot k_4)\Big\{2\ \alpha_{24}+\frac{\alpha_{34}\rho}{\alpha_{12}}+\frac{\rho^2}{\alpha_{12}}+\frac{{\alpha_{23}}^2}{\alpha_{34}}+\\
&+\frac{{\alpha_{24}}^2}{\alpha_{34}}+\frac{2\ \alpha_{23}\,\alpha_{24}}{\alpha_{34}}+\frac{\alpha_{23}\,\alpha_{12}}{\alpha_{34}}+\frac{\alpha_{12}\,\alpha_{24}}{\alpha_{34}}\Big\}-\\
&-(\zeta_2\cdot k_4)(\zeta_3\cdot k_1)\Big\{-\rho-2\ \alpha_{23}-2\ \alpha_{34}-\alpha_{12}-\alpha_{24}\Big\}\Big)\Big\}\Big]+\\
+(\text{cyclic permutations of indexes }&(1,2,3,4,5))\,. \qquad\qquad (\text{B.2})
\end{aligned}
$$

In the appendix D we compute the same amplitudes, using the standard Feynman

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diagram technique, from the action (6.1) and verify explicitly that both approaches give the same result.

C. The Feynman rules

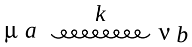
In this appendix we derive the Feynman rules from the lagrangian $\mathcal{L} = \mathcal{L}_{(0,2)} + \mathcal{L}_{(3)}$, where $\mathcal{L}_{(0,2)}$ and $\mathcal{L}_{(3)}$ are given in (4.1) and (4.3), respectively. We will follow the standard procedure employed in non-abelian gauge theories. The gauge fields are matrices in the U(n) internal space, so that $A_\mu = A_\mu^a \lambda^a$, where the hermitian generators λ^a satisfy the usual relations

$$\mathrm{tr}\,\lambda^a \lambda^b = \delta^{ab}\,; \qquad \left[\lambda^a, \lambda^b\right] = i\, f^{abc}\, \lambda^c\,. \tag{C.1}$$

Adding the gauge fixing term

$$\mathcal{L}_{GF} = -\frac{1}{2\alpha}\,\mathrm{tr}(\partial_\mu A^\mu)^2 \tag{C.2}$$

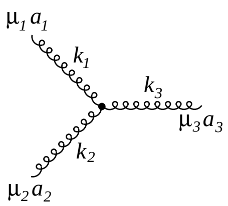
and choosing the *Feynman gauge* condition $\alpha = 1$, we obtain from eq. (4.1) the propagator



$$: \delta^{ab}\,\frac{\eta_{\mu\nu}}{k^2}\,, \tag{C.3}$$

where $k^2 = -k_0^2 + \vec{k}^2$, in agreement with the convention for $\eta_{\mu\nu}$, given in (2.6). Since we are only interested in the tree level amplitudes, it will not be necessary to include *ghost fields*.

The three point-vertex comes only from the standard F^2 term in eq. (4.1) and is given by



$$\begin{aligned}
: \mathcal{V}^{a_1 a_2 a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3) = \\
-i\, g\, f^{a_1 a_2 a_3} \Big[(k_1 - k_2)_{\mu_3}\, \eta_{\mu_1\mu_2} + \\
+ (k_2 - k_3)_{\mu_1}\, \eta_{\mu_2\mu_3} + (k_3 - k_1)_{\mu_2}\, \eta_{\mu_1\mu_3} \Big] \\
\equiv g\, f^{a_1 a_2 a_3}\, V_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)\,.
\end{aligned} \tag{C.4}$$

Our momentum conventions in the previous vertex as well as in the following ones are such that the momenta are all inwards. Also, when computing an N gluon amplitude, factors $-i\,(2\pi)^{10}\,\delta^{10}(k_1 + k_2 + \cdots + k_N)$ and $i/(2\pi)^{10}$ are to be included for each interaction vertex and internal lines, respectively. In some steps of the calculation, it will be convenient to employ the relations (C.1) so that we can write (C.4) as follows

$$\mathcal{V}^{a_1 a_2 a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3) = -g \sum_{\mathrm{perm}'} \mathrm{tr}\,(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}) \left[\left(k_{1\,\mu_3}\,\eta_{\mu_1\mu_2} - k_{1\,\mu_2}\,\eta_{\mu_1\mu_3} \right) + \text{cyclic perm} \right] \tag{C.5}$$

where the meaning of $\sum_{\mathrm{perm}'}$ is similar to the one in eq. (2.1) (except that here we have Lorentz indices instead of polarizations) and the terms in *cyclic perm* are obtained adding

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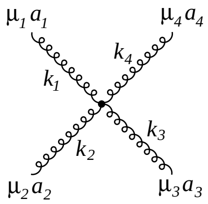
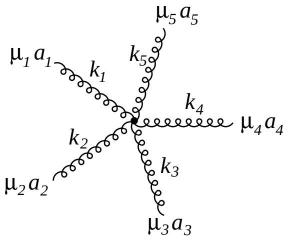
the cyclic permutations of the Lorentz indices and momenta. The four and five gluon vertices can be expressed similarly as follows



$$: \mathcal{V}^{a_1 a_2 a_3 a_4}_{\mu_1 \mu_2 \mu_3 \mu_4}(k_1, k_2, k_3, k_4) = g^2 \sum_{\text{perm}'} \text{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4}) V_{\mu_1 \mu_2 \mu_3 \mu_4}(k_1, k_2, k_3, k_4) \tag{C.6}$$

and



$$: \mathcal{V}^{a_1 a_2 \dots a_5}_{\mu_1 \dots \mu_5}(k_1, \dots, k_5) = g^3 \sum_{\text{perm}'} \text{tr}(\lambda^{a_1}\lambda^{a_2} \dots \lambda^{a_5}) V_{\mu_1 \mu_2 \dots \mu_5}(k_1, k_2, \dots, k_5)\,, \tag{C.7}$$

where the Lorentz factors can be derived from (4.1) and (4.3) in a straightforward way. In the following we will present the results for the terms of order zero, two and three in α'. Let us begin with the four gluon vertex. The zeroth order contribution can be easily obtained from the first term in (4.1) and is given by

$$V^{(0)}_{\mu_1 \mu_2 \mu_3 \mu_4} = -\frac{1}{2}\left(\eta_{\mu_1 \mu_2}\, \eta_{\mu_3 \mu_4} - \eta_{\mu_1 \mu_3}\, \eta_{\mu_2 \mu_4}\right) + \text{cyclic perm.} \tag{C.8}$$

The terms proportional to α'^2 are also straightforward, but much more involved. We have employed computer algebra to extract this and all higher order vertex contributions. Exploring the tensor symmetry we obtain the following result

$$\begin{aligned}
&\left(\frac{6}{\pi^2 \alpha'^2}\right) V^{(2)}_{\mu_1 \dots \mu_4}(k_1 \dots k_4) = \\
&\quad \Big\{ \Big[\left(k_2 \cdot k_3\, \eta_{\mu_3 \mu_4} - k_{2\mu_3} k_{3\mu_4}\right) \left(k_1 \cdot k_4\, \eta_{\mu_1 \mu_2} - k_{4\mu_1} k_{1\mu_2}\right) + \\
&\qquad + (k_1, \mu_1)\,(k_4, \mu_4) \longleftrightarrow (\mu_1, k_1)\,(\mu_4, k_4) \Big] + \\
&\qquad + \Big[(k_1, \mu_1)\,(k_2, \mu_2)\,(k_3, \mu_3)\,(k_4, \mu_4) \longleftrightarrow (\mu_1, k_1)\,(\mu_2, k_2)\,(\mu_3, k_3)\,(\mu_4, k_4) \Big] \Big\} + \\
&\quad + 2 \times \{\text{previous curly bracket with } (\mu_3, k_3) \longleftrightarrow (\mu_4, k_4)\} - \\
&\quad - 2 \left\{ \left(k_1 \cdot k_2\, \eta_{\mu_1 \mu_2} - k_{1\mu_2} k_{2\mu_1}\right) \left(k_3 \cdot k_4\, \eta_{\mu_3 \mu_4} - k_{3\mu_4} k_{4\mu_3}\right) \right\} + \\
&\quad + \frac{1}{2} \times \{\text{previous curly bracket with } (\mu_2, k_2) \longleftrightarrow (\mu_3, k_3)\} + \\
&\quad + \text{cyclic perm,}
\end{aligned} \tag{C.9}$$

where an interchange like $(k_1, \mu_1)\,(k_2, \mu_2) \longleftrightarrow (\mu_1, k_1)\,(\mu_2, k_2)$ yields, for instance, $k_1 \cdot k_3 \longleftrightarrow k_{3\mu_1}$ and $k_1 \cdot k_2 \longleftrightarrow \eta_{\mu_1 \mu_2}$. In writing the above expression, we have organized the

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terms in such a way that there is a direct correspondence with the terms of order α'^2 in eq. (4.1). The second and the third term of eq. (4.1) generates the first and the second curly brackets of eq. (C.9), respectively. The fourth and the fifth terms of eq. (4.1) generates the the last two curly brackets eq. (C.9).

Let us now consider the contributions to $V_{\mu_1\ldots\mu_4}(k_1\ldots k_4)$ which are of order α'^3. These are generated from the last five terms of eq. (4.3) with the covariant derivative replaced by the usual one. These terms can also be written, exploring the tensor symmetry, as follows

$$\begin{aligned}
&\left(\frac{1}{\zeta(3)\alpha'^3}\right) V^{(3)}_{\mu_1\ldots\mu_4}(k_1\ldots k_4) = \\
&\Big\{16\Big[\left(k_{2\mu_1}k_1\cdot k_3 - k_{3\mu_1}k_1\cdot k_2\right)\left(\eta_{\mu_2\mu_3}\left(k_3\cdot k_4 k_{2\mu_4} - k_2\cdot k_4 k_{3\mu_4}\right) - \mu_2 \longleftrightarrow k_2\right) + \\
&\qquad + \left(\eta_{\mu_1\mu_3}k_1\cdot k_2 - k_{2\mu_1}k_{1\mu_3}\right)\left(k_{3\mu_4}\left(k_{4\mu_2}k_2\cdot k_3 - k_{3\mu_2}k_2\cdot k_4\right) - \mu_4 \longleftrightarrow k_4\right)\Big]\Big\} + \\
&+ \Big\{8\, k_1\cdot k_2\Big[\left(\eta_{\mu_3\mu_4}\, k_2\cdot k_4 - k_{2\mu_4}\, k_{4\mu_3}\right)\left(\eta_{\mu_1\mu_2}\, k_1\cdot k_3 - k_{3\mu_1}\, k_{1\mu_2}\right) + \\
&\qquad\qquad + (k_1,\mu_1)\,(k_3,\mu_3) \longleftrightarrow (\mu_1,k_1)\,(\mu_3,k_3)\Big] + \\
&\qquad + \left[(k_1,\mu_1)\,(k_2,\mu_2)\,(k_3,\mu_3)\,(k_4,\mu_4) \longleftrightarrow (\mu_1,k_1)\,(\mu_2,k_2)\,(\mu_3,k_3)\,(\mu_4,k_4)\right]\Big\} + \\
&+ \left\{\text{previous curly bracket with } (\mu_2,k_2) \longleftrightarrow (\mu_3,k_3)\right\} - \\
&- \left\{8\, k_1\cdot k_3\left[\left(k_2\cdot k_4\eta_{\mu_2\mu_4} - k_{4\mu_2}\, k_{2\mu_4}\right)\left(k_1\cdot k_3\eta_{\mu_1\mu_3} - k_{1\mu_3}\, k_{3\mu_1}\right)\right]\right\} + \\
&+ \left\{16\, k_1\cdot k_4\left(\eta_{\mu_1\mu_3}k_1\cdot k_3 - k_{1\mu_3}k_{3\mu_1}\right)\left(\eta_{\mu_2\mu_4}k_2\cdot k_3 - k_{3\mu_2}k_{2\mu_4}\right) - \mu_4 \longleftrightarrow k_4\right\} + \\
&+ \text{cyclic perm}\,. \qquad\qquad (C.10)
\end{aligned}$$

Similarly to the eq. (C.9), each one of the five brackets in the previous expression is in direct correspondence with the last five terms in eq. (4.3). The compact and symmetric form of these expressions makes then quite useful when performing the calculations of scattering amplitudes. Everything can be expressed in terms of the product of two tensors of the form of the zeroth order four-gluon vertex given by (C.8).

An important property of the four-gluon vertices in eq. (C.9) and (C.10) is that they obey simple Ward identities like

$$\begin{aligned}
k_1^{\mu_1}\, V^{(2,3)}_{\mu_1\ldots\mu_4}(k_1\ldots k_4) &= k_2^{\mu_2}\, V^{(2,3)}_{\mu_1\ldots\mu_4}(k_1\ldots k_4) = k_3^{\mu_3}\, V^{(2,3)}_{\mu_1\ldots\mu_4}(k_1\ldots k_4) = \\
&= k_4^{\mu_4}\, V^{(2,3)}_{\mu_1\ldots\mu_4}(k_1\ldots k_4) = 0\,. \qquad (C.11)
\end{aligned}$$

These identities are a direct consequence of the invariance of the Born-Infeld action under a non-abelian gauge transformation

$$A_\mu \longrightarrow A_\mu - \frac{1}{g}\partial_\mu\omega + i\,[A_\mu,\omega]\,, \qquad (C.12)$$

where ω is an infinitesimal matrix. The *zero* on the right hand side of eq. (C.11) reflects the absence of A^3 terms of order α'^2 or higher. In fact, since each individual term in the lagrangians (4.1) and (4.3) is gauge invariant by itself, the identity (C.11) is fulfilled by each curly bracket in eqs. (C.9) and (C.10).

The contribution of order α'^2 to the five-gluon vertex can also be written in a very compact form as follows

$$
\begin{aligned}
&\left(\frac{6}{4\pi^2\alpha'^2}\right) V^{(2)}_{\mu_1\ldots\mu_5}(k_1\ldots k_5) = \\
&\Big\{\Big[\left(k_1\cdot k_5\eta_{\mu_1\mu_2} - k_{1\mu_2}k_{5\mu_1}\right)\left(k_{2\mu_3}\eta_{\mu_4\mu_5} - k_{2\mu_4}\eta_{\mu_3\mu_5}\right) + \\
&\quad + (\mu_2,k_2)\leftrightarrow(\mu_5,k_5)\Big] - \mu_5\leftrightarrow k_5\Big\} + \\
&+\Big\{\left[\left(k_1\cdot k_4\eta_{\mu_1\mu_2} - k_{1\mu_2}k_{4\mu_1}\right)\left(k_{5\mu_4}\eta_{\mu_3\mu_5} - k_{5\mu_3}\eta_{\mu_4\mu_5}\right) - (\mu_2,k_2)\leftrightarrow(\mu_3,k_3)\right] + \\
&\quad + \left[\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3} - \eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right)\left(k_{3\mu_5}k_4\cdot k_5 - k_{4\mu_5}k_3\cdot k_5\right) - \mu_3\leftrightarrow k_3\right] + \\
&\quad - \mu_4\leftrightarrow k_4\Big\} + \\
&+\left\{\left(k_{1\mu_3}\eta_{\mu_1\mu_2} - k_{1\mu_2}\eta_{\mu_1\mu_3}\right)\left(k_4\cdot k_5\eta_{\mu_4\mu_5} - k_{4\mu_5}k_{5\mu_4}\right) - (\mu_1,k_1)\leftrightarrow(\mu_3,k_3)\right\} + \\
&+\left\{\left(k_1\cdot k_3\eta_{\mu_1\mu_3} - k_{1\mu_3}k_{3\mu_1}\right)\left(k_{2\mu_5}\eta_{\mu_2\mu_4} - k_{2\mu_4}\eta_{\mu_2\mu_5}\right)\right\} + \\
&+ \text{cyclic perm.}
\end{aligned}
\tag{C.13}
$$

Notice that after including all the cyclic permutations this expression generates 300 terms.

We have verified that (C.13) and (C.9) are related by the following Ward identity

$$
k_1{}^{\mu_1} V^{(2)}_{\mu_1\ldots\mu_5}(k_1,\ldots,k_5) = V^{(2)}_{\mu_2\mu_3\mu_4\mu_5}(k_1+k_2,k_3,k_4,k_5) - V^{(2)}_{\mu_2\mu_3\mu_4\mu_5}(k_2,k_3,k_4,k_1+k_5),
\tag{C.14}
$$

which is again a consequence of the gauge invariance of the effective action (4.1) under the gauge transformation (C.12).

Finally, the contributions of order α'^3 to the five gluon vertex are given by

$$
\begin{aligned}
&\left(\frac{1}{16\zeta(3)\alpha'^3}\right) V^{(3)}_{\mu_1\ldots\mu_5}(k_1\ldots k_5) = \\
&\Big\{\left[(k_{1\mu_4}\eta_{\mu_1\mu_2} - k_{1\mu_2}\eta_{\mu_1\mu_4})(k_4\cdot k_5k_{3\mu_5} - k_3\cdot k_5k_{4\mu_5})k_{2\mu_3} + (\mu_4,k_4)(\mu_5,k_5)\leftrightarrow(k_4,\mu_4)(k_5,\mu_5)\right] + \\
&\quad + \left[(k_{3\mu_5}\eta_{\mu_2\mu_3} - k_{3\mu_2}\eta_{\mu_3\mu_5})(k_4\cdot k_5\eta_{\mu_1\mu_4} - k_{4\mu_1}k_{5\mu_4})k_1\cdot k_2 + (\mu_1,k_1)(\mu_4,k_4)\leftrightarrow(k_1,\mu_1)(k_4,\mu_4)\right] + \\
&\quad + (\mu_3,k_3)(\mu_5,k_5)\leftrightarrow(k_3,\mu_3)(k_5,\mu_5)\Big\}_{(1)} + \\
&+\Big\{\text{previous curly bracket with } (\mu_1,k_1)(\mu_2,k_2)(\mu_3,k_3)(\mu_4,k_4)(\mu_5,k_5)\leftrightarrow \\
&\qquad \leftrightarrow(\mu_1,k_1)(\mu_3,k_3)(\mu_5,k_5)(\mu_4,k_4)(\mu_2,k_2)\Big\}_{(2)} + \\
&+\left\{\left[\left(k_1\cdot k_2\eta_{\mu_1\mu_4} - k_{1\mu_4}k_{2\mu_1}\right)\left(k_3\cdot k_5\eta_{\mu_3\mu_5} - k_{3\mu_5}k_{5\mu_3}\right)k_{4\mu_2} - (\mu_2,k_2)\leftrightarrow(\mu_4,k_4)\right] - \mu_4\leftrightarrow k_4\right\}_{(3)} + \\
&+\Big\{\left[\left(k_1\cdot k_2\eta_{\mu_1\mu_4} - k_{1\mu_4}k_{2\mu_1}\right)\left(k_2\cdot k_4k_{5\mu_2} - k_2\cdot k_5k_{4\mu_2}\right)\eta_{\mu_3\mu_5} - \mu_5\leftrightarrow k_5\right] + \\
&\quad + \left[\left(k_{5\mu_4}\eta_{\mu_2\mu_5} - \eta_{\mu_4\mu_5}k_{5\mu_2}\right)\left(k_2\cdot k_3\eta_{\mu_1\mu_3} - k_{2\mu_3}k_{3\mu_1}\right)k_1\cdot k_2 - \mu_1\leftrightarrow k_1\right] + \\
&\quad + \left[\left[\left(k_{5\mu_3}\eta_{\mu_2\mu_5} - \eta_{\mu_3\mu_5}k_{5\mu_2}\right)k_{2\mu_4} - \mu_2\leftrightarrow k_2\right]\left(k_1\cdot k_2k_{4\mu_1} - k_1\cdot k_4k_{2\mu_1}\right)\right] + \\
&\quad + \left[\left(k_2\cdot k_5\eta_{\mu_4\mu_5} - k_{2\mu_5}k_{5\mu_4}\right)\left(k_1\cdot k_2\eta_{\mu_1\mu_3} - k_{1\mu_3}k_{2\mu_1}\right)k_{3\mu_2} - \mu_3\leftrightarrow k_3\right] + \\
&\quad + \left[\left[\left(k_1\cdot k_4\eta_{\mu_1\mu_2} - k_{1\mu_2}k_{4\mu_1}\right)\left(k_{4\mu_5}k_{5\mu_3} - \eta_{\mu_3\mu_5}k_4\cdot k_5\right)k_{3\mu_4} - \mu_3\leftrightarrow k_3\right] - (\mu_2,k_2)\leftrightarrow(\mu_3,k_3)\right] + \\
&\quad + \left[\left[\left(k_{1\mu_4}\eta_{\mu_1\mu_3} - k_{1\mu_3}\eta_{\mu_1\mu_4}\right)\left(k_{4\mu_2}k_{2\mu_5} - \eta_{\mu_2\mu_5}k_2\cdot k_4\right)k_4\cdot k_5 - \mu_5\leftrightarrow k_5\right] - (\mu_2,k_2)\leftrightarrow(\mu_3,k_3)\right] + \\
&\quad + \left[\left[\left(k_1\cdot k_2\eta_{\mu_1\mu_3} - k_{1\mu_3}k_{2\mu_1}\right)\left(k_{3\mu_2}k_{2\mu_5} - k_2\cdot k_3\eta_{\mu_2\mu_5}\right) - \mu_3\leftrightarrow k_3\right]k_{3\mu_4} - (\mu_4,k_4)\leftrightarrow(\mu_5,k_5)\right] - \\
&\quad - \left[\left(k_{2\mu_5}\eta_{\mu_2\mu_3} - k_{2\mu_3}\eta_{\mu_2\mu_5}\right)\left(k_{1\mu_4}k_{2\mu_1} - k_1\cdot k_2\eta_{\mu_1\mu_4}\right)k_5\cdot(k_4+k_3) - \mu_5\leftrightarrow k_5\right] +
\end{aligned}
$$

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$$
\begin{aligned}
&+\big[\big[\big(k_4\cdot k_5\eta_{\mu_2\mu_5}-k_{4\,\mu_5}k_{5\,\mu_2}\big)\big(k_1\cdot k_4\eta_{\mu_3\mu_4}-k_{1\,\mu_4}k_{4\,\mu_3}\big)-(\mu_3,k_3)\leftrightarrow(\mu_2,k_2)\big]\,k_{2\,\mu_1}-\mu_1\leftrightarrow k_1\big]+\\
&+\big[\big[\big(\eta_{\mu_3\mu_5}k_4\cdot k_5-k_{4\,\mu_5}k_{5\,\mu_3}\big)\big(k_{4\,\mu_2}k_{3\,\mu_4}-\eta_{\mu_2\mu_4}k_3\cdot k_4\big)-\mu_3\leftrightarrow k_3\big]\,k_{3\,\mu_1}-(\mu_1,k_1)\leftrightarrow(\mu_2,k_2)\big]+\\
&+\big[\big(k_1\cdot k_2\eta_{\mu_1\mu_3}-k_{1\,\mu_3}k_{2\,\mu_1}\big)\,\big(k_2\cdot k_5\eta_{\mu_2\mu_4}-k_{2\,\mu_4}k_{5\,\mu_2}\big)\,k_{3\,\mu_5}-\mu_5\leftrightarrow k_5\big]+\\
&+\big[\big(k_2\cdot k_5k_{4\,\mu_5}-k_4\cdot k_5k_{2\,\mu_5}\big)\,\eta_{\mu_2\mu_4}-\mu_2\leftrightarrow k_2\big]\,\big(k_1\cdot k_2\eta_{\mu_1\mu_3}-k_{1\,\mu_3}k_{2\,\mu_1}\big)+\\
&+\big[\big(k_{4\,\mu_1}k_1\cdot k_3-k_{3\,\mu_1}k_1\cdot k_4\big)\,\eta_{\mu_2\mu_4}-\mu_4\leftrightarrow k_4\big]\,\big(k_4\cdot k_5\eta_{\mu_3\mu_5}-k_{4\,\mu_5}k_{5\,\mu_3}\big)+\\
&+\big[\big(k_4\cdot k_5\eta_{\mu_2\mu_5}-k_{4\,\mu_5}k_{5\,\mu_2}\big)\,\big(k_1\cdot k_4\eta_{\mu_3\mu_4}-k_{1\,\mu_4}k_{4\,\mu_3}\big)\,k_{3\,\mu_1}-\mu_1\leftrightarrow k_1\big]\Big\}_{(4)}+\\
+\frac{1}{2}\Big\{&\big[\big[\big[\big(k_{3\,\mu_2}\eta_{\mu_1\mu_3}-\eta_{\mu_2\mu_3}k_{3\,\mu_1}\big)\,\big(k_{2\,\mu_4}k_{1\,\mu_5}-k_{2\,\mu_5}k_{1\,\mu_4}\big)-(\mu_3,k_3)\leftrightarrow(\mu_5,k_5)\big]-\\
&\quad-\mu_2\leftrightarrow k_2\big]k_2\cdot k_3-\mu_1\leftrightarrow k_1\big]+\\
&\quad+\big[\big[\big(\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}-\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}\big)\big(k_1\cdot k_4k_{5\,\mu_1}-k_1\cdot k_5k_{4\,\mu_1}\big)-\mu_4\leftrightarrow k_4\big]k_4\cdot(k_2+k_3)-\mu_5\leftrightarrow k_5\big]+\\
&\quad+\big[\big[\big(\eta_{\mu_2\mu_5}k_{5\,\mu_3}-k_{5\,\mu_2}\eta_{\mu_3\mu_5}\big)\,\big(k_3\cdot k_4\eta_{\mu_1\mu_4}-k_{3\,\mu_4}k_{4\,\mu_1}\big)+(\mu_4,k_4)\leftrightarrow(\mu_5,k_5)\big]\,k_3\cdot k_4-\\
&\quad-(\mu_1,k_1)\leftrightarrow(\mu_2,k_2)\big]+\\
&\quad+\big[\big[\big(k_{5\,\mu_2}\eta_{\mu_1\mu_5}-\eta_{\mu_2\mu_5}k_{5\,\mu_1}\big)\,\big(k_{1\,\mu_4}k_{2\,\mu_3}-k_{1\,\mu_3}k_{2\,\mu_4}\big)-\mu_2\leftrightarrow k_2\big]\,k_2\cdot k_4-\mu_1\leftrightarrow k_1\big]+\\
&\quad+\big[\big[\big(k_2\cdot k_3k_{1\,\mu_3}-k_{2\,\mu_3}k_1\cdot k_3\big)\,\big(\eta_{\mu_2\mu_5}k_{5\,\mu_1}-k_{5\,\mu_2}\eta_{\mu_1\mu_5}\big)-\mu_2\leftrightarrow k_2\big]\,k_{2\,\mu_4}-\mu_1\leftrightarrow k_1\big]+\\
&\quad+\big[\big[\big(k_{5\,\mu_1}\eta_{\mu_3\mu_5}-\eta_{\mu_1\mu_5}k_{5\,\mu_3}\big)\,\big(k_{3\,\mu_4}k_1\cdot k_4-k_3\cdot k_4k_{1\,\mu_4}\big)-\mu_3\leftrightarrow k_3\big]\,k_{4\,\mu_2}-\mu_1\leftrightarrow k_1\big]+\\
&\quad+\Big[\big[\big(k_{4\,\mu_1}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_4}k_{4\,\mu_2}\big)\,\big(k_1\cdot k_5k_{2\,\mu_5}-k_{1\,\mu_5}k_2\cdot k_5\big)-\mu_2\leftrightarrow k_2\big]\,(k_2-k_4)_{\mu_3}-\mu_1\leftrightarrow k_1\Big]\Big\}_{(5)}+\\
+\Big\{&\big[\big[\big(\eta_{\mu_2\mu_5}\eta_{\mu_1\mu_4}-\eta_{\mu_2\mu_4}\eta_{\mu_1\mu_5}\big)\,\big(k_1\cdot k_3k_{2\,\mu_3}-k_{1\,\mu_3}k_2\cdot k_3\big)-\mu_2\leftrightarrow k_2\big]\,k_2\cdot(k_4+k_5)-\mu_1\leftrightarrow k_1\big]+\\
&+\big[\big[\big(k_1\cdot k_5\eta_{\mu_1\mu_4}-k_{1\,\mu_4}k_{5\,\mu_1}\big)\,\big(k_{2\,\mu_3}\eta_{\mu_2\mu_5}-k_{2\,\mu_5}\eta_{\mu_2\mu_3}\big)-\mu_5\leftrightarrow k_5\big]\,k_2\cdot k_5-(\mu_3,k_3)\leftrightarrow(\mu_4,k_4)\big]+\\
&+\big[\big[\big(k_{1\,\mu_5}k_{3\,\mu_1}-k_1\cdot k_3\eta_{\mu_1\mu_5}\big)\,\big(k_{5\,\mu_2}k_{2\,\mu_3}-\eta_{\mu_2\mu_3}k_2\cdot k_5\big)-\mu_3\leftrightarrow k_3\big]-\mu_5\leftrightarrow k_5\big]\,k_{2\,\mu_4}-\\
&-\big[\text{previous bracket with }(\mu_1,k_1)(\mu_2,k_2)(\mu_3,k_3)(\mu_4,k_4)(\mu_5,k_5)\leftrightarrow\\
&\quad\leftrightarrow(\mu_4,k_4)(\mu_3,k_3)(\mu_1,k_1)(\mu_5,k_5)(\mu_2,k_2)\big]\,k_{2\,\mu_5}\Big\}_{(6)}+\\
+\Big\{&\big[\big(k_2\cdot k_4\eta_{\mu_2\mu_4}-k_{2\,\mu_4}k_{4\,\mu_2}\big)\,k_{2\,\mu_5}-(\mu_4,k_4)\leftrightarrow(\mu_5,k_5)+\big(k_{2\,\mu_5}\eta_{\mu_2\mu_4}-k_{2\,\mu_4}\eta_{\mu_2\mu_5}\big)\,(k_4+k_5)\cdot k_2\big]\times\\
&\times\big(k_1\cdot k_3\eta_{\mu_1\mu_3}-k_{1\,\mu_3}k_{3\,\mu_1}\big)+\big(k_2\cdot k_5\eta_{\mu_2\mu_5}-k_{2\,\mu_5}k_{5\,\mu_2}\big)\,\big(k_{1\,\mu_4}\eta_{\mu_1\mu_3}-k_{1\,\mu_3}\eta_{\mu_1\mu_4}\big)\,k_2\cdot k_5\Big\}_{(7)}+\\
+\Big\{&\big[\big[\big[\big[\big(k_{3\,\mu_5}\eta_{\mu_1\mu_3}-\eta_{\mu_3\mu_5}k_{3\,\mu_1}\big)\,\big(k_{2\,\mu_4}k_{4\,\mu_2}-k_2\cdot k_4\eta_{\mu_2,\mu_4}\big)+(\mu_4,k_4)(\mu_5,k_5)\leftrightarrow(\mu_4,\mu_5)(k_5,\mu_5)\big]-\\
&\quad-(\mu_4,k_4)\leftrightarrow(\mu_5,k_5)\big]k_1\cdot k_2+\big(k_3\cdot k_5\eta_{\mu_3\mu_5}-k_{3\,\mu_5}k_{5\,\mu_3}\big)\,\big(k_4\cdot k_5\eta_{\mu_1\mu_4}-k_{5\,\mu_4}k_{4\,\mu_1}\big)\,k_{1\,\mu_2}\big]-\\
&\quad-\mu_1\leftrightarrow k_1\big]+\\
&+\big[\big[\big(k_{5\,\mu_2}\eta_{\mu_3\mu_5}-\eta_{\mu_2\mu_5}k_{5\,\mu_3}\big)\,\big[\big(k_1\cdot k_2\eta_{\mu_1\mu_4}-k_{1\,\mu_4}k_{2\,\mu_1}\big)+\big(k_1\cdot k_3\eta_{\mu_1\mu_4}-k_{1\,\mu_4}k_{3\,\mu_1}\big)\big]+\\
&+\big(k_{1\,\mu_4}\eta_{\mu_1\mu_3}-k_{1\,\mu_3}\eta_{\mu_1\mu_4}\big)\,\big(k_2\cdot k_5\eta_{\mu_2\mu_5}-k_{2\,\mu_5}k_{5\,\mu_2}\big)\big]k_4\cdot k_5-\mu_4\leftrightarrow k_4\big]+\\
&+2\big[\big(k_2\cdot k_5\eta_{\mu_2\mu_5}-k_{2\,\mu_5}k_{5\,\mu_2}\big)\big(\big(k_1\cdot k_2\eta_{\mu_1\mu_4}-k_{1\,\mu_4}k_{2\,\mu_1}\big)\,k_{5\,\mu_3}-\big(k_1\cdot k_2\eta_{\mu_1\mu_3}-k_{1\,\mu_3}k_{2\,\mu_1}\big)\,k_{5\,\mu_4}\big)\big]\Big\}_{(8)}+\\
+\,&\text{cyclic perm.}
\end{aligned}
\tag{C.15}
$$

Notice that we have numbered the curly brackets in the previous expression in correspondence with the terms in the lagrangian (4.3). The F^5 terms in (4.3) yields the first three curly brackets in eq. (C.15). Since there are no four-gluon vertex associated with these terms, the corresponding Ward identities are quite simple insofar as its right hand side is identically zero. This simple property has been explicitly verified for the first three curly brackets in eq. (C.15). Each of the curly brackets numbered from (4) to (8) satisfy an Ward identity similar to the one in eq. (C.14). Therefore the complete vertex $V^{(3)}_{\mu_1,\ldots,\mu_5}(k_1,\ldots,k_5)$

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satisfy

$$k_1{}^{\mu_1}\, V^{(3)}_{\mu_1 ..\mu_5}(k_1,\ldots,k_5) = V^{(3)}_{\mu_2\mu_3\mu_4\mu_5}(k_1+k_2,k_3,k_4,k_5) - V^{(3)}_{\mu_2\mu_3\mu_4\mu_5}(k_2,k_3,k_4,k_1+k_5)\,. \tag{C.16}$$

D. The three, four and five-point amplitudes

We will now employ the Feynman rules presented in appendix C in order to compute the scattering amplitudes $\mathcal{A}^{(N)}$ of N gluons with colors $a_1, a_2, \ldots, a_N$, polarizations $\zeta_1, \zeta_2, \ldots, \zeta_N$ and external momenta $k_1, k_2, \ldots, k_N$, satisfying the physical conditions in (2.5). In what follows we will present and discuss the explicit results for $N = 3, 4, 5$ up to order α'^3.

The three-gluon amplitude $\mathcal{A}^{(3)}$ can be easily obtained from eq. (C.5). Contracting with $\zeta_{1\mu_1}\,\zeta_{2\mu_2}\,\zeta_{3\mu_3}$ and inserting the factor $-i\,(2\pi)^{10}\,\delta^{10}(k_1+k_2+k_3)$ one easily obtains

$$\mathcal{A}^{(3)} = i\,g\,(2\pi)^{10}\,\delta^{10}(k_1+k_2+k_3)\sum_{\text{perm}} \text{tr}\,(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3})\,[(\zeta_3\cdot k_1)\;(\zeta_1\cdot\zeta_2) - (\zeta_2\cdot k_1)\;(\zeta_1\cdot\zeta_3)]\,. \tag{D.1}$$

Using the cyclic invariance of the trace we can write eq. (D.1) as follows

$$\mathcal{A}^{(3)} = i\,(2\pi)^{10}\,\delta^{10}(k_1+k_2+k_3)\sum_{\text{perm}'} \text{tr}\,(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3})\,A(1,2,3)\,, \tag{D.2}$$

where

$$A(1,2,3) = g\;[(\zeta_3\cdot k_1)\,(\zeta_1\cdot\zeta_2) - (\zeta_2\cdot k_1)\,(\zeta_1\cdot\zeta_3)] + (\text{cyclic perm}) \tag{D.3}$$

As we can see, this is the simplest example were general structure of the amplitudes given in eq. (2.1) arises. Inserting eq. (D.3) into eq. (D.2) one can easily obtain the result given in eq. (3.2). There are some simple properties of (D.2) which is shared by any other amplitude. First of all, it exhibits the Bose symmetry under permutations of the external gluons. Secondly, it is invariant under Lorentz transformations. And last but not least, it is invariant under independent gauge transformations $\zeta_i \to \zeta_i + \alpha_i\,k_i$, $(i = 1,2,3)$ of the 3 external fields. One should also remark that (D.2) is an exact tree amplitude independent of the parameter α'. That would not be so, if the lagrangian contained a term like $\alpha'\,F^3$.

Let us make some other observations about the three gluon amplitude. Making use of the gauge freedom $\zeta_i^{\mu_i} \to \zeta_i^{\mu_i} + \alpha_i\,k_i^{\mu_i}$, $(i = 1,2,3)$, one can always choose a gauge such that all the time components of $\zeta_i^{\mu_i}$, $(i = 1,2,3)$ are equal to zero. In this case, the transversality condition implies that $\vec{\zeta}_i \perp \vec{k}_i$, $(i = 1,2,3)$. On the other hand, the physical conditions on the external momenta implies that $\vec{k}_1\,||\,\vec{k}_2$ and $\vec{k}_1\,||\,\vec{k}_3$. Therefore, the polarizations of three the external gluons are all perpendicular to $\vec{k}_1$, $\vec{k}_2$ and $\vec{k}_3$ and the amplitude (D.2) vanishes for this choice of gauge. Since it is gauge invariant, it is always zero. Physically this is just a consequence of the conservation of the total (spin and orbital) angular momentum of the three on-shell transverse massless particles. Our strategy to the computation of the four and five gluon amplitudes will be first to derive the relation (2.1) and then to compute the Lorentz scalars $A(1,2,3,4)$ and $A(1,2,3,4,5)$. As we will see, these Lorentz

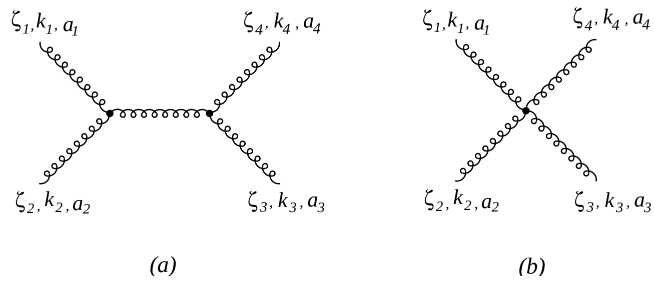

Figure 2: Basic tree diagrams which contributes to the four gluon scattering amplitude. There are three permutations of the diagram (a) which can be obtained fixing the external gluon ($\zeta_1\, k_1\, a_1$) and cyclic permuting the other three.

scalars can be expressed in terms combinations of contractions of the Lorentz scalars in the Feynman rules. All we need to do that are a few simple relations involving the color factors associated with the diagrams shown in figures 2 and 3.

The color factor associated with the diagram (a) of figure 2 can be expressed in terms of traces of the group generators as

$$C_{(4)}^{a_1 a_2 a_3 a_4} \equiv g^2\, f^{a_1 a_2 d}\, f^{d a_3 a_4} = -i\, g^2\, \mathrm{tr}\left(\lambda^{a_1}\,[\lambda^{a_2}, \lambda^d]\right) f^{d a_3 a_4} = -g^2\, \mathrm{tr}\,(\lambda^{a_1}\,[\lambda^{a_2}, [\lambda^{a_3}, \lambda^{a_4}]])\,, \tag{D.4}$$

where we have employed eq. (C.1). Similarly, the diagram (a) of figure 3 has a color factor

$$\begin{aligned} C_{(5a)}^{a_1 a_2 a_3 a_4 a_5} \equiv g^3\, f^{a_1 a_2 d}\, f^{d e a_5}\, f^{a_3 a_4 e} &= -i\, g^3\ \mathrm{tr}\left(\lambda^{a_1}\,[\lambda^{a_2}, \lambda^d]\right)\, f^{d e a_5}\, f^{a_3 a_4 e} \\ &= -g^3\ \mathrm{tr}\,(\lambda^{a_1}\,[\lambda^{a_2}, [\lambda^e, \lambda^{a_5}]])\ f^{a_3 a_4 e} \\ &= i\, g^3\ \mathrm{tr}\,(\lambda^{a_1}\,[\lambda^{a_2}, [[\lambda^{a_3}, \lambda^{a_4}], \lambda^{a_5}]])\,. \end{aligned} \tag{D.5}$$

The diagram (b) of figure 3 can be expressed in terms of six different color factors, each one corresponding to the different traces in the four gluon vertex given by eq. (C.6). For instance, one these factors is

$$C_{(5b)}^{a_1 a_2 a_3 a_4 a_5} \equiv g^3\ \mathrm{tr}\,(\lambda^{a_1}\,\lambda^{a_2}\,\lambda^{a_3}\,\lambda^e)\, f^{e a_4 a_5} = -i\, g^3\ \mathrm{tr}\,(\lambda^{a_1}\,\lambda^{a_2}\,\lambda^{a_3}\,[\lambda^{a_4}, \lambda^{a_5}]) \tag{D.6}$$

and the other five are obtained by doing permutations of $\{a_2, a_3, e\}$ and replacing λ^e by $[\lambda^{a_4}, \lambda^{a_5}]$. We have now the basic ingredients and definitions in order to generate the Lorentz scalars $A(1,2,3,4)$ and $A(1,2,3,4,5)$.

Using the relation (D.4) in the expression associated with the diagram (a) of figure 2 and performing the permutations as indicated in the figure, we obtain for the Lorentz factor of $\mathrm{tr}\,(\lambda^{a_1}\,\lambda^{a_2}\,\lambda^{a_3}\,\lambda^{a_4})$ the following expression

$$-A(1,2,3,4) = V_{33}(1,2,3,4) - V_{33}(1,4,2,3) - V_4(1,2,3,4)\,. \tag{D.7}$$

The explicit form of the Lorentz scalar V_{33} is given by

$$V_{33}(1,2,3,4) = \zeta_1^{\mu_1}\,\zeta_2^{\mu_2}\,\zeta_3^{\mu_3}\,\zeta_4^{\mu_4}\ \left.\frac{V_{\mu_1 \mu_2 \sigma}(k_1, k_2, -k_1 - k_2)V^{\sigma}{}_{\mu_3 \mu_4}(k_1 + k_2, k_3, k_4)}{2\, k_1 \cdot k_2}\right|_{\mathrm{phys}}\,, \tag{D.8}$$

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where V is the Lorentz factor of the three gluon vertex as defined in eq. (C.4) and the subscript "phys" means that the conditions (2.5) are being used. The scalar V_4 is given by

$$V_4(1,2,3,4) \,=\, \zeta_1^{\mu_1}\,\zeta_2^{\mu_2}\,\zeta_3^{\mu_3}\,\zeta_4^{\mu_4}\Big(V^{(0)}_{\mu_1\ldots\mu_4}(k_1,\ldots,k_4)+V^{(2)}_{\mu_1\ldots\mu_4}(k_1,\ldots,k_4)+ \\ +V^{(3)}_{\mu_1\ldots\mu_4}(k_1,\ldots,k_4)\Big)\Big|_{\rm phys}\,. \tag{D.9}$$

Making a systematic use of the physical conditions (2.5), we were able to simplify the result to the following form:

$$-\,A(1,2,3,4) \,=\, g^2\left(-\frac{2}{(k_1\cdot k_2)\,(k_1\cdot k_4)}+\frac{4\,\pi^2}{3}\,\alpha'^2+16\,(k_1\cdot k_3)\zeta(3)\,\alpha'^3\right)\times \\ \times\,K(\zeta_1,k_1;\zeta_2,k_2;\zeta_3,k_3;\zeta_4,k_4) \tag{D.10}$$

where the kinematic factor $K(\zeta_1,k_1;\zeta_2,k_2;\zeta_3,k_3;\zeta_4,k_4)$ agrees with (3.6). Finally, inserting the factors $-i\,(2\pi)^{10}\delta^{10}(k_1+k_2+k_3+k_4)$, the color factor $\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4}$ and adding the remaining non-cyclic permutations, we obtain

$$\mathcal{A}^{(4)}=i\,(2\pi)^{10}\,\delta^{10}(k_1+k_2+k_3+k_4)\sum_{\rm perm'}{\rm tr}\,(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4})\,A(1,2,3,4)\,. \tag{D.11}$$

As we can see, eq. (D.11) exhibits the structure of the general amplitude given by eq. (2.1). The on-shell gauge invariance of $\mathcal{A}^{(4)}$ can be argued by the following two nice properties satisfied by the kinematic factor K present in (D.10):[11] it has *total* symmetry in the four external particles (so K may be written as a common factor in (D.11)) and it vanishes whenever any ζ_i is substituted by the corresponding k_i, after using the physical conditions in (2.5) (so K is, itself, on-shell gauge invariant). We remark that the α' corrections are associated only with the quartic vertex in figure 2 (b).

Let us now consider the five gluon amplitude shown in figure 3. Using eqs. (D.5) and (D.6), performing the permutations indicated in the figure 3 and collecting the coefficient of ${\rm tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4}\lambda^{a_5})$, we obtain the following expression for the Lorentz scalar $A(1,2,3,4,5)$

$$A(1,2,3,4,5) \,=\, i\Big[V_{333}(1,2,3,4,5)+V_{333}(1,5,3,4,2)+ \\ +V_{333}(1,2,5,4,3)+V_{333}(5,4,2,3,1)-V_{333}(1,5,2,3,4)- \\ -V_{43;1}(1,2,3,4,5)-V_{43;1}(3,4,5,1,2)- \\ -V_{43;3}(1,4,5,2,3)-V_{43;6}(1,2,5,3,4)+V_{43;3}(4,2,3,1,5)+ \\ +V_5(1,2,3,4,5)\Big]_{\rm phys}\,, \tag{D.12}$$

where

$$V_{333}(1,2,3,4,5) = \\ \frac{V_{\mu_1\mu_2}{}^{\rho}(k_1,k_2,-k_1-k_2)V^{\sigma}{}_{\mu_3\mu_4}(-k_3-k_4,k_3,k_4)V_{\mu_5\rho\sigma}(k_5,k_1+k_2,k_3+k_4)}{(2\,k_1\cdot k_2)(2\,k_3\cdot k_4)}\times$$

[11] See [13, section 7.4.3].

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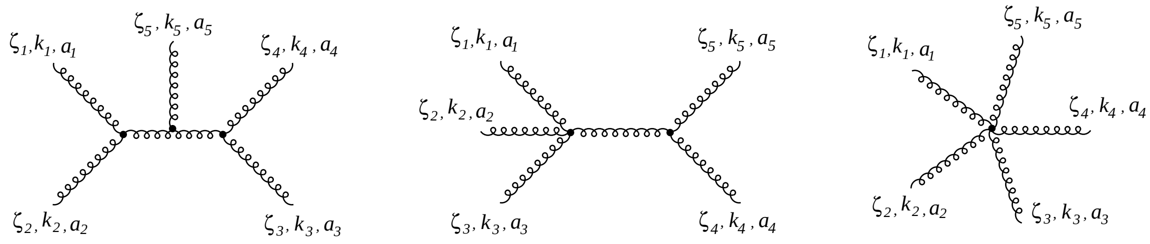

(a) *(b)* *(c)*

Figure 3: Basic tree diagrams which contributes to the five gluon scattering amplitude. The 15 permutations of the diagram (a) can be grouped in 5 sets of three permutations. The first set is obtained fixing the external gluons $(\zeta_1\, k_1\, a_1)$ and $(\zeta_5\, k_5\, a_5)$ and cyclic permuting the other three. The other four sets are obtained from the first performing $5 \leftrightarrow 1$, $5 \leftrightarrow 2$, $5 \leftrightarrow 3$ and $5 \leftrightarrow 4$. There are 10 permutations of diagram (b) which correspond to the 10 distinct possibilities $[(4,5), (3,5), (2,5), (1,5), (3,4), (2,4), (1,4), (2,3), (1,3), (1,2)]$ for the external gluons in the cubic vertex

$$\times\, \zeta_1^{\mu_1}\, \zeta_2^{\mu_2}\, \zeta_3^{\mu_3}\, \zeta_4^{\mu_4}\, \zeta_5^{\mu_5}\Big|_{\rm phys}, \tag{D.13}$$

$$\begin{aligned} V_{43;i}(1,2,3,4,5) =& \\ &\left[\frac{V^{(0)}_{(i)\,\mu_1\mu_2\mu_3}{}^{\rho}(k_1,k_2,k_3,k_4+k_5)V_{\rho\mu_4\mu_5}(-k_4-k_5,k_4,k_5)}{(2\,k_4\cdot k_5)} + \right.\\ &+\frac{V^{(2)}_{(i)\,\mu_1\mu_2\mu_3}{}^{\rho}(k_1,k_2,k_3,k_4+k_5)V_{\rho\mu_4\mu_5}(-k_4-k_5,k_4,k_5)}{(2\,k_4\cdot k_5)} + \\ &\left.+\frac{V^{(3)}_{(i)\,\mu_1\mu_2\mu_3}{}^{\rho}(k_1,k_2,k_3,k_4+k_5)V_{\rho\mu_4\mu_5}(-k_4-k_5,k_4,k_5)}{(2\,k_4\cdot k_5)}\right] \times \\ &\times\, \zeta_1^{\mu_1}\, \zeta_2^{\mu_2}\, \zeta_3^{\mu_3}\, \zeta_4^{\mu_4}\, \zeta_5^{\mu_5}\Big|_{\rm phys}; \qquad (i=1,\ldots,6) \end{aligned} \tag{D.14}$$

and

$$V_5(1,2,3,4,5) = \zeta_1^{\mu_1}\, \zeta_2^{\mu_2}\, \zeta_3^{\mu_3}\, \zeta_4^{\mu_4}\, \zeta_5^{\mu_5}\left(V^{(2)}_{\mu_1\ldots\mu_5}(k_1,\ldots,k_5) + V^{(3)}_{\mu_1\ldots\mu_5}(k_1,\ldots,k_5)\right)\Big|_{\rm phys}. \tag{D.15}$$

The quantities $V^{(0),(2),(3)}_{(i)\,\mu_1\mu_2\mu_3,\mu_4}$ $(i = 1\ldots 6)$ are the Lorentz factors of the independent traces (see eq. (C.6))

$$\begin{aligned} &{\rm tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4})\,,\\ &{\rm tr}(\lambda^{a_1}\lambda^{a_3}\lambda^{a_4}\lambda^{a_2})\,,\\ &{\rm tr}(\lambda^{a_1}\lambda^{a_4}\lambda^{a_2}\lambda^{a_3})\,,\\ &{\rm tr}(\lambda^{a_1}\lambda^{a_4}\lambda^{a_3}\lambda^{a_2})\,,\\ &{\rm tr}(\lambda^{a_1}\lambda^{a_3}\lambda^{a_2}\lambda^{a_4}) \end{aligned}$$

and

$${\rm tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_4}\lambda^{a_3})\,,$$

respectively.

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The eqs. (D.12) to (D.15), together with the results of the appendix C, constitute a well defined set of inputs to a computer aided calculation. The computation of these Lorentz scalars, as well as the ones associated with the four gluon amplitude, was performed using the *Maple* version of the computer algebra package HIP [11]. Although the result contains a large number of terms (the full expression, including the cyclic permutations, contains 562, 1272 and 8992 terms respectively for the powers 0, 2 and 3 of α'), we were able to verify that eq. (D.12) reproduces exactly the expression given in eq. (5.29). We have also explicitly verified the gauge invariance of the final amplitude

$$\mathcal{A}^{(5)} = i\,(2\pi)^{10}\,\delta^{10}(k_1+k_2+k_3+k_4+k_5)\sum_{\text{perm}'}\operatorname{tr}\left(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4}\lambda^{a_5}\right)A(1,2,3,4,5)\,. \quad \text{(D.16)}$$

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